Digital Compensation and Pendulum Clocks

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Introduction

This article explores the use of digital compensation for a pendulum clock.

Traditionally, pendulum clocks have had gears with integer ratios in order to convert periods or beats to seconds or minutes or astronomical cycles; they have dials with clutch adjustable hands in order to set the time-of-day; they have rating nuts or weight trays in order to adjust the rate; and they employ a variety of mechanical solutions in order to compensate for environmental effects such as slow changes in temperature, humidity, and pressure.

Like it or not, we live in an era where many simple, reliable, mechanical "analog" solutions are being replaced with complex, fragile, digital systems. The world of electronic timekeeping (quartz and atomic clocks), and UTC itself, provides an example of mathematical and digital adjustments to time and rate. Recent advances in low-cost, high-resolution MEMS sensors make real-time monitoring and active software compensation of pendulum clocks a possibility.

Let's see what happens if we combine sensors, equations, and digital time and rate adjustments to the operation of a pendulum clock.

This clock is 1 second fast

The photo on the left shows a Santa Fe Railroad telegraph operator in 1943:

The cropped image on the right shows the two pendulum clocks. It is interesting that they use two different clocks for two different time zones (Pacific and Mountain). But what really caught my attention was the sign that says "this clock is [1 second fast]". This is a fine example of keeping accurate time with a numerical correction instead of a physical adjustment.

Decoupling the pendulum from the clock display

It is easy to understand why placing a sign on a clock is easier than adjusting the time of the dial itself. Most pendulum clocks have a mechanical linkage between the pendulum and the clock face so adjusting by fractions of a period is problematic.

The situation is easier with pendulums that have an electrically impulsed clock dial. This would include vintage clocks such as Synchronome, Shortt, and Fedchenko as well as more modern pendulum clocks such as Littlemore. Because optical or magnetic beat sensing is so easy, most DIY pendulum designs separate the swinging pendulum *timekeeper* from the analog or digital *time display*, regardless if the display uses hour, minute, second hands (with mechanical gears) or uses HH:MM:SS digits (with electronic digital divider circuits).

In such a clock, it is possible to add an electrical *delay* between the pendulum and the clock. Thus, instead of placing a placard on the clock that says "fast by 1 second", one could just delay the electrical pulse coming from the pendulum by 1 second before it goes to the clock display. There are many ways to delay an electrical signal by a fixed or variable amount and also ways to accommodate both positive (fast) and negative (slow) delays.

This clock is ¼ second a day fast

The photo on the left is a marine chronometer. On the right is a typical rate calibration chart:

TABLE I

COMPUTATION OF RATE

Date	Dial Error + = Fast - = Slow		Daily Rate + = Gain - = Loss	Mean Deviation in Daily Rate	Remarks
	Min	Sec			
Oct 1948					
3	+0	2			Started+Set
4	+0	2½	+½		
5	+0	2½	0	¼	
6	+0	3	+½	¼	
7	+0	3	0	¼	
8	+0	3½	+½	¼	
9	–	–	—	—	Not wound
10	+0	4	+¼	—	2 day avg.

(Mean daily rate = +1/4 second)

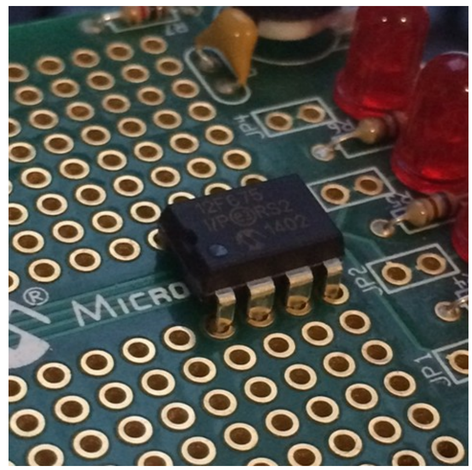
In Table I, there will be noted a column headed "Mean Deviation in Daily Rate." The

It is well-known that a clock doesn't have to keep *perfect* rate as long as you are confident what the *error* in rate is. Once the rate error is quantified (accuracy) and trusted (stability) you simply apply a linear rate correction to every time reading of the clock. In this example you would progressively subtract an additional ¼ second per day since October 3, 1948. Of course, having a chronometer that was perfectly "on-time" and "on-rate" would be ideal, but it is not necessary as long as you have a calibration chart, pencil, and paper.

The first photo was a 1 second *time* correction; the second was a ¼ second/day *rate* correction. In neither case is the clock itself adjusted. The correction is done manually by the user, since the clock mechanism runs undisturbed.

A programmable *time adjustment* chip

There are many ways to delay a pulse by a fraction of a second: long wires, relays, RC circuits, acoustic delay lines, and one-shot multivibrators come to mind. But the easiest is digital delays. Since the electronics world is full of precise timing it should not surprise you that it is trivial to construct a digital IC board or program a microcontroller so that it delays a pulse. The photo shows a typical "PIC" microcontroller. The chip is less than ½×½ inch in size.



Most pendulum clocks are not more accurate than a microsecond. In fact, neither is the widely used Microset timer. So nanosecond accuracy is not needed. The design I have in mind would allow the user to configure the chip (using a RS232 serial port) to delay anywhere from 0 to 1 million microseconds; that's sufficient to cover one beat of a standard 1 meter pendulum.

The idea is that the chip would be installed between the pendulum and the clock display. The chip has one input: it waits for a pulse from the pendulum (usually an optical beat signal). The chip has one output: the input pulse delayed by a specific number of microseconds. This pulse then goes straight to the analog or digital clock.

It may seem silly to want to delay the beat of a pendulum – but this would allow the clock display to "tick" at precisely the correct moment (e.g., relative to UTC), even if the pendulum bob itself does not. Decimal switches or a PC could be used to send the correct 6 digit delay number to the chip. The pendulum would continue to swing *out of phase* but the clock would now show the correct precise time, even at the millisecond or microsecond level.

A programmable *rate adjustment* chip

If a chip can add a delay to each pulse and if you can tell the chip what that delay should be then imagine what happens if you tell the chip a slightly *different* delay number *every* second? The net result is a change in rate. For example, if the delay starts at 0 and each second the delay increases by 2.9 µs (microseconds) then by the end of the day (86400 s) the delay is 0.250 s, or ¼ second. Another way to look at it: if the input frequency is 86400.25 / 86400 = 1.000002893 Hz then the output frequency is 1.000000000 Hz.

This means if a pendulum is fast by ¼ second/day and you apply this 2.9 µs incrementing delay to each pulse from the pendulum, the output of the chip (which is the input to the clock) will be on-time. Therefore, this is a way to change pendulum rate without touching the pendulum. By dialing-in the precise amount of rate error, the chip creates the correct rate for the clock. The pendulum continues to swing at its incorrect rate but the clock display is now running at the correct precise rate.

There is no magic here and nothing to be suspicious about. The chip simply acts like a gear train. One rate goes in and one rate goes out. Neither gears nor the chip add or subtract accuracy, they just change rate. The advantage of using digital circuits is that its "gears" are frictionless and the number of its "teeth" is essentially unlimited.

I have used this concept already in a *sidereal time* chip. Using the digital incrementing delay technique I created sidereal seconds from standard seconds. In this case the "gear" ratio is basically 86201 / 86400 (i.e., the ratio of sidereal day to solar day). Any rational number can be implemented. For greater accuracy the chip I developed (PD29) uses 861,640,916 / 864,000,000. This represents a rate difference of 2730 ppm.

The advantages of a time and rate chip should now be more obvious. This means one can adjust the rate of a pendulum clock without touching the pendulum; without stopping, restarting and waiting hours or days for it to re-stabilize. The time and rate information is sent to the chip once, and from that point on, the clock runs at the desired rate and keeps the correct time.

With this chip, there is no need for rating nuts or weight trays. The pendulum can be any length that is convenient or optimal or "close enough". The time and rate compensation is done in software. Note that by using multiple chips a single pendulum can drive multiple clocks; for example, both standard and sidereal time.

The requirement that the pendulum have a 1 second beat is no longer absolute. The chip could be flexible enough to accommodate almost any input rate and still generate a 1 second output rate with the help of "digital gears". I have never believed that a 1.0 meter pendulum keeps the best time. Why not 25 cm or 80 cm or 125 cm or 250 cm? By uncoupling the pendulum rate from the clock rate one can now choose a pendulum length that gives the best performance and let the rate change chip take care of the clock. While the clock must tick in standard seconds, the pendulum itself does not.

Temperature compensation

What else can you do when you have a programmable rate chip between the pendulum and the clock? To me the first challenge would be temperature compensation.

The proposal is to continuously measure the ambient temperature with precise accurate digital sensor(s), compute the corresponding predicted change in pendulum rate, and then send that prediction to the rate chip. The net result is temperature compensation in the time shown by the clock, even though the pendulum itself is left uncompensated.

There are a number of advantages with this scheme. We already use temperature sensors to measure a pendulum's coefficient of temperature. Right now people take those results and design hardware to add to their pendulum bob as compensation. Instead why not just keep the sensors running and apply the calculated coefficients and algorithms to the running clock? Yes, you can tell I'm a software engineer.

One advantage is that temperature compensation is often more complex than a single linear equation. There are different parts to the clock. There are gradients. Different metals have different expansion coefficients, thermal mass, and heat transfer characteristics. All this can be modeled with equations. One can use high order polynomials or power series or equations that take time constants into account. So it seems to me that adaptive software can do a far more thorough compensation than a single compensating metal washer can.

Humidity and pressure compensation

If software compensation for temperature works, then there is no reason we can't do the same for humidity and pressure as well. Like temperature, the readings from these sensors can also be converted to rate corrections, summed, and applied to the rate chip to slightly speed up or slow down the clock (not the pendulum) as compensation.

Another advantage to software compensation is that any interaction among measurements can be handled. With a full set of environmental sensors recording data every few seconds, the best possible algorithm can be devised which takes all readings into account. Moreover, all this data can be logged so it is possible to change algorithms and "play back" the raw pendulum and sensor data to see how well one algorithm compares with another. All this can be done while the unadjusted pendulum continues to swing.

Software compensated pendulum clock

So now you understand what I mean by a digital, or software compensated pendulum clock. The pendulum is constructed as simple as possible and runs without any mechanical compensation.

An array of environmental sensors is permanently placed on or near the clock and data is collected in real-time. A (tiny microcontroller or PC) computer program takes the data and computes the best possible prediction of rate error. That prediction is then feed into a rate change chip and the output of the chip is used to drive the clock. The pendulum (which you probably don’t see) keeps imperfect time because it is uncompensated. The clock (which you do see) keeps near perfect time because it incorporates all measurable compensation.

The question is – will using a complex system of sensors and mathematics and digital corrections result in better timekeeping than simple mechanical compensation to the pendulum itself? I don't know. But it seems worth a try. There is something satisfying about letting a pendulum run completely free and instead adjusting the time that comes out of it using sensors and calculated corrections.

More compensation ideas

We can digitally set the time and adjust the rate. Perhaps also temperature, humidity, and pressure can be compensated. But once the infrastructure is in place to use sensors and compensate in software, many other ideas come to mind:

1. Sidereal time – As mentioned earlier one can implement different time scales using a time/rate chip instead of gears. The same pendulum can be used for many different clocks. In general, a rational number rate shifting chip enables a wide variety of astronomical clock displays.

2. Altitude – Gravity varies with altitude. One could dial-in the elevation above sea level and the chip would adjust the clock rate accordingly. Use GPS to obtain altitude (please ignore that GPS also gives precise UTC, making pendulum clocks obsolete).

3. Amplitude – Circular error is always a concern with precision pendulum clocks. It is usually solved by choosing exceptionally low amplitude, or using an isochronous suspension, or controlling the amplitude by varying impulse, or using cheeks to alter the profile of the swing.

   But if amplitude can be accurately measured with some sensor then a software compensated pendulum clock can just eliminate circular error through a simple equation. The pendulum itself continues to swing as physics dictates, but the clock now shows no circular error.

4. Velocity – If there are rate effects due to variation in air damping or in energy then some sort of compensation can be added using measured velocity as a parameter.

5. Tilt – If a pendulum clock exhibits systematic rate variations due to tilt, then a measurement of tilt can be added to the list of software compensations.

6. Tides – The astronomical equations of earth-sun-moon are well known. If the pendulum performs so well that tides are visible then the rate compensation chip could easily "back out" tidal effects.

7. Acceleration (1D or 3D vibration, shock, microseisms) – This is a stretch but LIGO does this to compensate its sensitive suspended mirrors.

8. Period – Clearly, this would be going a *measurement too far*. Measuring period and adjusting rate to compensate for slightly too low or too high period is cheating. The clock then becomes the measurement (or is it the other way around) and this isn't fair.

I realize that a software compensated pendulum clock might end up looking less like a piece of kinetic art and more like a patient in the E.R. – with sensors and probes all over its body, surrounded by charts and graphs and computers.

There are limits to software compensation just as there are limits to mechanical compensation. Errors in pendulum timekeeping are both systematic and random – which is just another way to say that 1) some perturbations can be anticipated, detected, modeled, and partially compensated and, 2) some cannot.

Random perturbations are intrinsic to the design of the pendulum and the environment in which it operates. No amount of clever hardware or software compensation can fix flaws in design or installation.

The goal here is to analyze all the raw measurement data and extract as many systematic effects as possible. The resulting model is not likely a simple set of independent linear equations, but a complex mix of higher-order equations that take both instantaneous readings and reading history and interaction among sensors into account. During each beat the model predicts some infinitesimal amount of time or rate error and this is feed to the time/rate chip so that the clock keeps as perfect time as possible. Meanwhile, the pendulum swings freely, experiencing the full effect of everything in its environment.

Conclusion

An alternative method of pendulum rate compensation is possible when the "tick" of the pendulum is separate from the "tick" of the clock display. In this case it is possible to insert a programmable digital rate changer chip, which acts like a gear train with an arbitrary number of teeth.

The idea is that precision digital sensors make continuous environmental measurements. These are combined with coefficients and algorithms to best predict the effect on pendulum rate. This prediction is then sent to a "rate change chip" so that the clock displays compensated time.

The pendulum itself runs free and is not compensated in any way; not for phase, not for rate, not for temperature, humidity, or pressure, or even amplitude.

This scheme should result in more precise timekeeping since the compensating algorithms can be as complex as necessary to maximize the information in the environmental measurements.

I cannot say with certainty how well this will work or even if it's worth the effort. If nothing else, I hope the idea is stimulating. If there is sufficient interest within the horological community I will complete the design of the pendulum time and rate adjustment chip.

Further reading

- My open source PIC frequency dividers, digital timers, and sidereal rate generators:

  http://www.leapsecond.com/pic (full list of chips)
  http://www.leapsecond.com/pic/src/pd28.asm (sidereal)
  http://www.leapsecond.com/pic/src/pd29.asm (sidereal, high-precision)

- Here's an interesting device that converts solar time to sidereal time:

  A Solar-Sidereal Synchronous Motor (SZSD-1) for Driving Quartz Clocks
  http://adsabs.harvard.edu/full/1961SvA.....4..868S
  by Smirnov, E. I., Stepanov, V. S., & Tovchigrechko, S. S.

- The telegraph station photo appeared on eBay; a high-resolution image is here:

  One Second Fast: 1943
  http://www.shorpy.com/node/6229
  March 1943. "Seligman, Arizona. Teletype operator in the telegraph office of the Atchison, Topeka, and Santa Fe Railroad. The time here changes from Mountain to Pacific time." Medium-format safety negative by Jack Delano.

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