Understanding Hybrid Quartz / Pendulum Clocks

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Introduction

In recent decades it has been popular to enhance pendulum clocks with modern electronics: optical sensors, digital electronics, quartz oscillators, computer/microprocessors and adaptively controlled magnetic impulse. I was once suspicious of incorporating modern quartz crystal oscillators into traditional free pendulum clocks. This article shares my current understanding on the matter.

Perhaps you, like me, have wondered: Can a free pendulum clock keep better time if one uses electronics to aid its operation? If so, is it still a free pendulum clock? Is it fair to mix quartz and pendulum in this way? Are there different levels of mixing? Could adding a quartz oscillator or microprocessor accidentally degrade pendulum clock performance? With modern sensors where does one draw the line between monitoring and control? At what point is quartz simply driving the pendulum? Should pendulum clocks incorporating oscillators into their design have a different name?

Hybrid Technology

I recall a recent discussion about the merits of combustion, electric, and hybrid vehicles. Some people get very passionate when defending their own views on the matter. I can certainly see both sides of the pure-gasoline and pure-electric argument. As an engineer I also appreciate the complexity and ingenuity of the compromise hybrid solution. I thought: this is not unlike quartz controlled pendulum clocks.

GPS provides an even better example. We are all familiar with precision quartz crystal oscillators, known for their high stability and a wide selection of frequencies. Whether small or large, old or new, all quartz oscillators drift a little over time but their key attribute is the ability to generate very stable medium to high frequency signals, such as 32.768 kHz or 10 MHz.

Many of you know about a very clever technology called GPSDO, or GPS Disciplined Oscillator, which is the marriage of a highly stable quartz oscillator and a GPS timing receiver. Inside a typical GPSDO a 10 MHz quartz oscillator is compared against a GPS pulse once per second and if the quartz is gradually observed to drift it is slowly corrected.

Now quartz alone will freely drift and GPS alone does not produce 10 MHz. Combine the two and you get a drift-less 10 MHz signal. Lose the GPS signal and it simply becomes pure quartz again. This hybrid technology has revolutionized the commercial precision timing world.

So with these two examples it occurred to me that when one mixes quartz and pendulum a sort of *hybrid quartz-pendulum clock* is created. It is neither purely quartz, nor purely free pendulum; instead it is a useful combination of the two. While they differ significantly in fundamental frequency and Q, both are examples of simple harmonic oscillators, of time-keepers, of "ratekeepers". The question is how much quartz and how much pendulum is the final creation? What are the characteristics of these new breed of pendulum clocks? Are there any hidden dangers, assumptions or requirements?

Modern Electronics and Sensors

The ability (or shall we say, the temptation) to control pendulum clocks arises from the ability to measure pendulum clocks with unprecedented levels of precision.

Modern electronics has made this easy. What was impossible a century ago is now simple. Nanometers and microseconds are accessible. Who has not used a *MicroSet Timer*? You don't have to wonder how your pendulum is swinging, you can actually measure it, to very high precision, in real-time. You don't have to wait a days or months to see how many tenths of seconds you have gained or lost. Instead, you can see how many microseconds of error you have in mere minutes. You can plot, trend, and perform FFT in real-time. This new world of precision creates some interesting temptations and wonderful opportunities for amateur pendulum clock developers.

Accurate digital sensors with computer logging are available for every physical quantity: temperature, humidity, pressure, amplitude, velocity, acceleration, tilt, strain. Not to mention voltage, current, and frequency. It is possible to use lasers to measure changes in rod length, or gravimeters to measure changes in gravity, or seismometers to monitor ground motion. Most importantly, all these sensors operate with *zero* contact with the pendulum itself. The pendulum can run freely; it has absolutely no idea it is being monitored with intensive care.

The data one gets from this entire symphony of sensors and statistical analysis is invaluable to uncover flaws or subtle deficiencies in pendulum design. It allows one to more rapidly analyze, improve, and then validate modified designs. It should speed up pendulum clock evolution and make it easier to match the classic free pendulum clocks of the 1900's.

Passive Monitor or Active Control

If your pendulum is 37 microseconds ahead, what can you do to correct it? What if you notice that its amplitude is 25 arcseconds too wide? Suppose you detect the rod length has grown 2.7 nanometers since last week. If your sensors tell you local gravity has decreased by 15 microgals, how can you compensate? What should you do if you see a trend where the period is 9.2 parts per billion too fast?

The ability to measure with great resolution in real-time gives us detailed information that no one had back in the golden age of pendulum clocks. But what do we do with the information. How do we use this new power? Do we try to understand *why* the pendulum is deviating unexpectedly or do we just beat the pendulum into submission?

This is the crux of the philosophical debate over using modern electronics, especially oscillators, to improve the performance of pendulum clocks. Measuring the slightest errors in pendulum motion is not the issue. What we do with the information can stir debate.

A century ago the old masters did not have all this technology or this real-time data. I presume they spent months or years uncovering each little flaw in their designs. If pendulum rod length varied they spent time to figure out the root metallurgical cause. If pendulum amplitude varied too much they spent time studying friction and impulse, making prototypes, doing experiments, striving for more constant energy loss, constant energy gain, constant amplitude by inherent design. They didn't just grab a microprocessor and make the amplitude problem "go away".

But perhaps I am being too harsh, because it is really exciting to try to use modern technology in creative new ways. I am as guilty as anyone. I simply think it's important that we admit that active control systems, based on stable quartz oscillators, create a different class of pendulum clock. It’s a class where one type of clock helps another type of clock.

Still, at the end of the day, when temperature is controlled, when velocity is controlled, when amplitude is controlled, when length is controlled, when period is now consistent, when timing is near perfect – don't you wonder deep down why they varied in the first place? Wouldn't you like to find out the root cause? Isn't it somehow more honorable to identify physical and engineering problems and cleverly reduce or eliminate them at a fundamental design level rather than simply measuring errors against a stable reference and adjusting the operation of the pendulum as a closed loop?

Degrees of Control

The main defense of hybrid pendulum clocks is that quartz is not "controlling" the pendulum. Yes, a word like *drive* or *control* may be too strong. Perhaps words like influence, correct, direct, manage, regulate, improve, monitor, or enhance better describe the subtle role that quartz plays in a typical hybrid pendulum clock. So let's not get too picky about the choice of word just yet.

A hybrid pendulum clock is one which combines two oscillators or timekeeping components in such a way that the net performance is improved. Specifically here we mean combining a quartz oscillator with a pendulum to create a more self-regulating, better-performing clock.

What does it mean for quartz to *control* a pendulum clock? When does control fade to mere *suggestion*, or gentle *guidance*, or gradual *adjustment*, or digital *assist*, or occasional *correction*, or continuous *error reduction*? The word control is problematic because it implies some sort of constant dictatorial force. I can see why most people object to describing their hybrid pendulum clocks as quartz-*controlled*-pendulum-clocks. In almost all cases the word control is just too strong and does injustice to the creative design.

Ten Shades of Gray

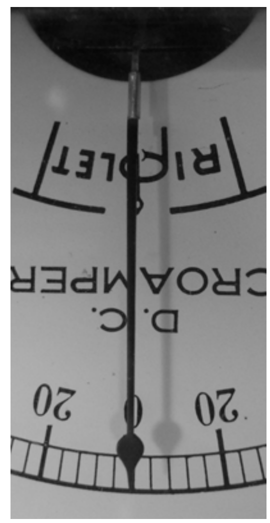
None of us are trying to create the best clock. If that were the goal, we would all have long since moved on to quartz or rubidium or cesium or GPS clocks. No, the goal has never been the best clock; it is the best *pendulum* clock. It is this self-imposed restriction that gives us pause when we add too much non-pendulum sensor and control technology and thus take away some of what makes a pendulum clock free.

The issue of quartz, control, and pendulum clocks is not black & white. There are many levels of control. There are many parameters that lend themselves to control: time, rate, energy, amplitude, velocity, length, and even gravity. To better understand the intersection of quartz and pendulum it is worth studying the gray area between pure quartz clocks (black) and pure pendulum clocks (white).

On the extreme white side we have all the classic free precision pendulum clocks. With no sensors and no quartz, there is no question about control. Note that Shortt and Riefler predate the invention of quartz oscillators (1930's). Accurate quartz oscillators were around when the Fedchenko clocks were developed but it is interesting to note he chose an ingenious isochronous suspension rather than quartz to reduce the effects of amplitude variation. Would Shortt have used quartz had it been invented back then? Why did Fedchenko not use quartz?

On the extreme black side we have ubiquitous quartz driven clocks without any pendulum at all. So what of the vast gray area in between these two extremes, the area where quartz is somehow mixed with pendulum? We will now examine 10 different hybrid pendulum clocks in the gray area, in approximate order from the most dependent on quartz to the least.

Gray 10 – Sinewave Driven



Imagine the most extreme hybrid pendulum clock; one that is so completely controlled and so totally dominated by quartz timekeeping, that it scarcely can be called a pendulum clock. To the right is a photo of such a clock; look closely and see the jewel and spring suspension at the top, the long tapered rod and aerodynamic bob against a calibrated amplitude scale on the back wall of the case. If the photo were a movie you would see it gently swinging back and forth. Do I hear you laugh yet?

Here's how it works: an accurate quartz oscillator is used to synthesize a slow low-voltage sinewave which drives the upside-down vintage analog center-reading panel meter. The lightweight thin black metallic *rod* and spade-shaped *bob* move exactly in sync with the phase, amplitude and rate of the sinewave.

In this admittedly contrived and humorous example of a hybrid pendulum clock the quartz is clearly totally responsible for the performance of the pendulum. This is one instance where there is no debate about using the word *control*, in the strictest sense of the word.

Note that the Q of the pendulum is very low, perhaps below 1. As a result the quartz oscillator is easily able to *drive* the pendulum with any period ranging from fractions of a second to many minutes or longer. The suspension damping is large. The combination of gravity and damping insure the bob rests at center when the quartz controller is inactive. Amplitude, velocity and period are controlled by the quartz waveform. There is no circular or escapement error.

Not surprisingly this pendulum clock has low temperature, humidity and barometric pressure sensitivities. The ovenized quartz oscillator (not shown) has a Q on the order of a million and so the pendulum clock keeps exceedingly good time. It is also quite immune from changes in acceleration of gravity due to tides and the clock does not require vacuum for best performance.

You should make one of these (the original Triplett-brand meter is seen on the left) as they are very entertaining, accurate, quiet and compact!

This hybrid pendulum clock is essentially 100% quartz and the pendulum is a mere puppet. It may win a prize for being accurate but a real pendulum clock needs to be free to march to its own beat.

Gray 9 – Pulse Driven

In the Triplett example above the quartz-synthesizer controlled the movement of the pendulum, both in space (angle) and in time. The power of the continuous magnetic field was sufficiently high or the Q of the pendulum was sufficiently low that it completely submitted to quartz timekeeping. The bob had no choice but to follow.

The next step towards freeing the pendulum is to increase its Q. With a higher Q pendulum, the continuous quartz sinewave drive is no longer necessary since the pendulum is capable of swinging by itself for some time. The quartz only needs to apply a periodic rather than continuous magnetic impulse.

We've probably all seen examples of this sort of hybrid pendulum clock; beautiful, simple, masterfully built. They also tend to keep "perfect" time because although the bob is completely free to swing on its own, the period locks to the quartz impulse rate. Without the impulse the pendulum would gradually come to rest (how gradually is a measure of Q). The clock will function correctly if the natural period of the pendulum is close enough to the impulse period created by the quartz oscillator. The pendulum is "captured" by the quartz.

Although the pendulum swings by itself, quartz is driving the time. Specifically we can say the short-term behavior of the clock is due to the pendulum but the long-term performance is determined by the quartz.

This clock is less dependent on quartz than the previous example: the pendulum swings mostly on its own and quartz is used to deliver only a short pulse every period instead of a continuous sinewave. Although the pendulum may diverge by small fractions of a second in the short-term, it will always follow the impulse timing of the quartz in the long-term. The time accuracy of the quartz determines the time accuracy of the pendulum.

Gray 8 – Pulse Driven N/M

With a moderately high-Q pendulum there is no need to impulse on every period. A traditional 1 meter pendulum clock has a beat of 1 second and a period of 2 seconds. A carefully designed pendulum will easily "lock" onto the impulse rate even if the rate is a multiple. For example, applying one impulse every ten seconds or one every minute should still work.

More generally, if the ratio between the natural period of the pendulum and the period of impulse delivered by quartz is a modest rational number the pendulum will tend to synchronize to the impulse rate. For example one could have 7 impulses from quartz for every 13 periods of pendulum. I'm not sure why one would do this other than for novelty or for research. The higher the numbers the more precise the pendulum needs to be to maintain lock, so there is some challenge and some upper limit of reliable operation. Achieving a ratio of 355:113, for example, would be a special charm.

Compared to the previous example this reduction in impulse rate doesn't fundamentally alter the use of quartz. But the lower the rate of impulse the less it appears quartz is the driving force behind this hybrid pendulum clock. In the extreme we could imagine an interesting high-Q pendulum clock that receives one (massive) impulse just once an hour. Would it be possible to construct such a clock? A clock where the hourly "chime" is actually the impulse that keeps it going for another hour?

Like the previous examples the time accuracy of the quartz determines the time accuracy of the pendulum. Quartz is not perfect: if its time drifts by microseconds and milliseconds over days and weeks this hybrid pendulum clock will drift also. One could replace or augment the quartz with a rubidium or cesium atomic clock, or radio controlled signal (e.g., DCF77, JJY, MSF, WWVB) or a GPS/GNSS receiver. In this case the pendulum clock would truly keep perfect time forever (don't mention leap seconds).

Circular Error as Rate Control

So far the hybrid clocks mentioned have been <u>driven</u>; that is, quartz determines the rate and the pendulum blindly follows. We now move on to a more subtle form of pendulum clock control, one in which quartz is used more as a guide or a reference rather than a driver. Instead of control by leading, it is control by adjusting.

To first order, the period of a pendulum clock is $T \approx 2\pi\sqrt{(L/g)}$ which implies constant period, T, as long as pendulum rod length, L, and earth's acceleration of gravity, g, are constant. In terms of real numbers, given an earth g of 9.807 m/s^2, and L of 1 meter, and amplitude of 1°, the period is 2.006 373 seconds.

Looking deeper the period is also a function of amplitude, θ, the half-angle of swing:

$$T \approx 2\pi \cdot \sqrt{\frac{L}{g}} \cdot \left(1 + \frac{1}{16}\theta^2 + \frac{11}{3072}\theta^4 + \frac{173}{737280}\theta^6 + \frac{22931}{1321205760}\theta^8 + \dots\right)$$

Taking circular error into account the period is actually 2.006 412 seconds, which is about 20 ppm (parts per million) longer. This in itself is not a problem with pendulum clocks; one can easily adjust the length to achieve the desired 2.000 000 second period.

The problem is that circular error is a non-linear function of amplitude. Thus any *change* in amplitude causes a small *change* in period. Again, using real numbers, when θ = 1° then T = 2.006 411 688 but when θ increases by 1%, so that θ = 1.01° then T = 2.006 412 456. This means near 1° that period changes by a factor of only about 1/25,000 relative to amplitude. The factor is much worse for larger angles.

Circular error is one of the main enemies of precision pendulum timekeeping and huge effort is expended by clockmakers to keep amplitude constant (and why the smaller the amplitude the better). With one style of hybrid pendulum clock, however, circular error can actually be a friend instead of an enemy. Consider the following two examples.

Gray 7 – Time Control with PLL

In all the previous examples of hybrid pendulum clocks there is no hiding that quartz, to one degree or another, is completely driving the clock. There was no concern with amplitude; the only important feature was that the pendulum period would lock to the impulse rate.

Consider now a finely constructed pendulum clock where the pendulum is free and a fixed impulse is delivered on each beat or period of the pendulum. With no quartz in sight this pendulum clock would continue to run freely and likely drift in time. Let's now convert it to a hybrid pendulum clock, but with a much less invasive use of quartz.

To improve accuracy we compare the pendulum clock time (that is, counting swings) against the quartz time (that is, counting cycles) to determine if the pendulum clock is ahead or behind the quartz in time. Because circular error exists we can use amplitude as a rate control mechanism. Changing amplitude will slightly change the rate. If there is a measured time error we can adjust the impulse energy so that the pendulum slows down or speeds up to correct for the rate and time error that we observed. In this way we can dynamically adjust the amplitude of the pendulum (which adjusts the rate of the pendulum) to keep its time error at a minimum. This implements a phase locked loop (PLL).

We see in this example that quartz is no longer directly driving the pendulum clock. Instead quartz is used more as a reference rather than an active driver. The pendulum is allowed more to run at its own rate. When errors in pendulum clock time are observed, circular error is used as a "knob" to adjust rate to reduce the time errors. Note that the adjustments do not need to be sudden; a wide variety of process control algorithms exist which can be finely tuned for optimal control.

Again, the time accuracy of the quartz determines the time accuracy of the pendulum. But it does so in a quieter, more hidden, way. It demonstrates using quartz as an internal reference for controlling the pendulum, rather than the active source of time that the pendulum follows.

Gray 6 – Rate Control with FLL

If comparing pendulum clock time against quartz time is too blatant we can further reduce the impact of quartz by using it as a frequency reference instead of a time reference. We follow the above example but instead of comparing time, we compare only rate. We use quartz to measure the rate of the pendulum clock (or the period, or average period).

If a rate error is observed then impulse energy is adjusted so that amplitude changes so that the error in rate is reduced. This implements a frequency locked loop (FLL). If implemented correctly the hybrid pendulum clock will maintain a correct average rate in spite of any mechanical or environmental disturbance or drift.

At first glance this may appear too similar to the above example. But it is quite different. The use of quartz here will guarantee that the average rate of the pendulum will be correct. But it turns out that this does not guarantee that the time of the pendulum will also be correct.

Time is integrated rate; time error is integrated rate error. And when you integrate rate error you get a time error that resembles random walk. In this case the relative freedom that the pendulum enjoys incurs some amount of unavoidable long-term time error. This is our first example where there is a trade-off between keeping perfect time and having some freedom. In our quest to reduce the influence of quartz we clearly will sacrifice some performance.

Quartz Accuracy & Stability

At this point it is worth talking more about quartz oscillators. For a clock two things can be accurate: one is the time and the other is the rate (a/k/a the "going" or frequency, which is inverse period). But what makes a good clock is neither accurate time nor accurate rate. The quality that distinguishes one clock from another is its *stability*, that is, how well it keeps its rate, second to second, hour to hour, week to week. If you have stability you can hope for accuracy, but accuracy without stability is fleeting.

The logic behind this is that time (what the dial shows) can usually be adjusted. In addition for most clocks, rate can also be adjusted. So what's really important is not the accuracy of the time or even the accuracy of the rate but how well it keeps time once the time and rate have been set. This is what we mean by stability. And quartz is famous for being exceptionally stable. To further reinforce this concept here is a diagram showing consistent cycles from a *stable* clock.

Like a ruler the more regular the time divisions (period), the better the clock:

|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|

By contrast the following *unstable* clock has very inconsistent periods:

|----|----|---|-----|---|--|---|-----|------|-----|------|--------|----|---|--|--|---|----|---|-----|

Here is a clock that stable but *inaccurate* (too fast):

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And similarly this clock is stable and inaccurate (too slow):

|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|

When you use an oscillator as a timebase, the more accurate the better, the more ticks the better, and the more stable the better. The stability of quartz allows it to serve as a reference for measurements. That could be rod length, it could be period, it could be velocity, or it could be amplitude. But no matter what parameter you measure, a precise measurement requires a stable reference.

Gray 5 – Constant Velocity

It seems like cheating to measure the time or rate of a pendulum clock directly and then adjust the rate, directly or indirectly, in order to improve the performance of the pendulum.

So continuing our quest to further free the pendulum from the influence of quartz, we consider controlling and maintaining constant energy instead of period. Consistent energy gives consistent amplitude which reduces circular error which makes period more consistent which improves timekeeping. The energy of a pendulum is represented by both the peak velocity (kinetic energy) and by peak amplitude (potential energy).

Measuring peak velocity at center swing is very easy so this is what most people do. Typically one uses an optical flag/gate and a high frequency quartz oscillator. The width of the gate is interesting. If the gate is very narrow the peak velocity measurement is close to true and accurate, but the low counts tend to make it imprecise. On the other hand if the gate is very wide the high counts make the measurement very precise, but it is a less accurate measure of peak velocity because it is now partly an average velocity. There is a no-win trade-off between accuracy of data and timeliness of data.

The accuracy of quartz is not important for velocity measurement or for impulse. The measurements are not typically converted from counts to "real" units like meters per second. The counts are arbitrary. The same is true for the duration if the impulse. What is important is the stability. The oscillator has to maintain highly regular oscillations during the velocity measurement and during the impulse and also in the interval between measurement and impulse.

If any sort of averaging is used then the stability has to last over the entire span of data averaging. This puts a larger burden on selecting the correct oscillator. If there is too much jitter or wander in the frequency of the quartz oscillator the impulse will not be as accurate as it should be. So this is why quartz is used; its stability is amazing; which is why hybrid pendulum clocks work so well. You need the stability of quartz in order to make these precise real-time measurement and the precise real-time impulses and quartz can meet this requirement.

If there is any doubt, substitute the good quartz timebase with a poor one and see what happens. Not only will the velocity measurement be inexact but the delivered impulse will also be inexact. The pendulum will receive a less than ideal compensating impulse. The worse the timebase is, the more error there will be. It is easy to completely ruin a fine hybrid pendulum clock if the quartz is not good enough.

There are other advantages to this type of hybrid pendulum clock. Quartz oscillators drift. Not by much, but over weeks and months the frequency can change by parts per million. Time error grows quadratically so quartz timekeepers have their limits. But frequency at worst grows linearly (in most cases it is logarithmic). In this hybrid clock the quartz is used as a frequency reference, not a time reference. If all it is doing is measuring velocity the drift will be insignificant.

Another advantage of this type of hybrid pendulum clock is that controlling amplitude requires much less precision than controlling period. As noted above (for 1° amplitude) changes in period have 25,000 times more effect on timekeeping than changes in amplitude. The quartz oscillator is being used to control amplitude, so it doesn't need to be nearly as stable as in the case where quartz is used to control period. One could probably use a simple XO instead of an OCXO (oven controlled crystal oscillator).

But of course one is still using quartz. It's just that this clever synergy between quartz reference and pendulum clock allows one to make the most out of quartz short-term and the most out of the pendulum long-term. Let it never be said that quartz isn't responsible for the better timing. It's a matter of *leverage*. Or to put it another way, to get 8 decimal places of period stability you only need 4 decimal places of amplitude stability.

Gray 4 – Constant Amplitude

The other way to maintain constant energy is to measure amplitude instead of velocity. Who wants to control velocity anyway? It's really amplitude that needs to be controlled so why not just measure amplitude directly. This can be done. In fact the great advantage of measuring amplitude is that the pendulum is moving at its very slowest at this point, giving maximum amount of time for the sensor to deduce the peak amplitude. Whether non-contact amplitude measurements involve a quartz oscillator, I don't know. It would also avoid a worry about velocity measurements.

Measuring peak velocity is typically accomplished by timing a narrow optical gap at the bottom of the pendulum swing. Since the gap has fixed width the velocity measurement is really just a timing measurement. If the pendulum has achieved constant amplitude the width of the swing and the width of the gap are a fixed ratio. So in some respects a peak velocity measurement is really just a fractional period measurement. If you don't believe this consider the thought experiment where the optical gap is gradually increased in width until it matches the full displacement of the bob. As the gap goes from zero width to full width of the swing, the readings you get go from 100% pure velocity to 100 % pure period and everything in between.

This means measuring velocity is sort of like measuring a fraction of the period. At least it's not as blatant as controlling 100% of the period, but still, you see here how quartz timing can sneak its way to pendulum timekeeping. Measuring amplitude directly, instead of indirectly using velocity as a proxy, would avoid this issue.

Quantization & Synchronization Effects

It is worth mentioning at this point a subtle problem with using quartz oscillators in pendulum clocks. When velocity or amplitude or period is measured with a digital or microprocessor-based sensor the resolution is usually determined by the quartz clock frequency. For example, if a 1 MHz clock is used the resolution will be 1 µs (microsecond). The same granularity applies when digital logic or a microprocessor is used to drive the magnetic impulse. In general, for input or for output, the start, stop, and duration will all be, and can only be, integer multiples of this basic time quantum. This is called quantization and for us it causes two problems.

Quantization can severely limit resolution. If some event is truly 123.45 µs but measured with 1 µs granularity the reading will be exactly 123, the accuracy limited to ±0.8% due to quantization. One has to pay attention to this possible loss of accuracy and the ramifications of inaccurate measurements. We assume, for example, that measurements of pendulum velocity are true. But the number we get is the true velocity *plus* the error in the sensor *plus* the error in clock stability *plus* the error due to quantization. If noisy velocity measurements are used to control impulse then a noisy impulse will result. This is not ideal.

A related unwelcome effect can occur when two independent clock sources interact. If one clock exhibits quantization it can have a synchronizing effect on the other clock. When events in a quartz clock occur always and only at exact 1 µs boundaries it can influence or gradually steer the pendulum to become closer. This form of phase or injection locking has been known since Huygens.

This is not to say any one particular hybrid pendulum clock suffers from these effects, but they are always a worry. The designer must prove, and the admirer must trust, that these effects are not present. This is one reason hybrid pendulum clocks are met with skepticism. It's really hard to uncover the facts and prove that no accidental "quartz time" is leaking into "pendulum time".

The more stable the quartz clock and pendulum clock the more likely this can occur. One solution is to use a less stable quartz clock, or deliberately add random non-Gaussian noise to the clock. Another solution is to literally turn off the quartz clock at least once every swing; it will restart with a different phase every time, and possibly a slightly different frequency. The quantization is still present, but not the synchronization.

Gray 3 – Analog Impulse

One way to make a hybrid pendulum clock freer is to make the impulse purely analog. This will avoid the excessive influence of quartz stability on measurement or accidental digital synchronization of impulse. That is, the start, stop, duration, and magnitude of the magnetic impulse could be completely removed from the quartz timebase or from the control of the computer.

I'm not sure how this would best be accomplished. One could imagine that the optical flag or gate could directly trigger the impulse without any quartz timing. The duration of the pulse would be timed by the motion of the bob instead of by a quartz reference.

In order to implement amplitude control there needs to be a way to control the energy of the impulse. So if one can't vary the duration one must be able to vary the power. One way is digital or analog control of impulse current.

The other way is having fixed duration, and fixed power, but modulating the presence or absence of pulses on each beat or period. On average this would accomplish the same as varying the power. But digital control over impulse current would likely provide higher precision than any form of pulse-width-modulation (PWM). PWM may introduce unwanted FM (frequency modulation) noise in the pendulum's spectrum.

In any event, using the quartz oscillator only for amplitude/velocity measurement and not for impulse generation removes yet another role of quartz in the hybrid pendulum clock. For those more in the pursuit of purity than performance this is a useful direction.

Gray 2 – Weak Control Algorithms

In the hybrid clocks mentioned so far, the control algorithms have not been described in any detail. There may be large opportunities, but also hidden ramifications here.

Consider for example a simple case of amplitude control. The trivial method is to make one measurement of amplitude and, based in that value, deliver the right amount of impulse. The quartz timebase is used for the measurement and for the delivery. In this case the accuracy of the timebase is not a high concern since it can be done as a ratio. The stability is more important for three reasons.

In order to make a measurement you want the timebase to be as consistent as possible *during* the velocity measurement. And you want the timebase to be as consistent as possible *during* the impulse delivery. But if using ratios you want the timebase to be as consistent as possible *between* the measurement and the delivery. If all this occurs within a fraction of a second, or even one second the constraints on quartz stability are not that high. Most any oscillator is sufficient.

But if one decides to use averaging or any other form of advanced algorithm (proportional, PID, Kalman) then the timebase needs to be stable not just over one period, but over the *entire span* of the averaging interval. This means a much higher standard of stability is required of the quartz than before. It also means the quartz you select for the job is playing a correspondingly more important role in the hybrid pendulum clock.

So while complex algorithms give better results they do so because they rely on a more stable timebase, over much longer intervals of time. I will not go into Allan deviation statistics here, but this is how we analyze GPSDO performance.

Gray 1 – Daily Amplitude Check

As a final example of reducing the role of quartz in a hybrid pendulum clock, of making a clock that is as close to free as possible, consider one where quartz is not used continuously but only rarely. Imagine a pendulum clock that is able, with analog design, to continue running well for hours or days or weeks. Clearly it will drift a little. Errors in velocity, amplitude, or length, or gravity will contribute to errors in timekeeping. Without an independent standard of calibration there is no way for the pendulum to know it is running fast or slow (rate error), if it is gaining or losing time (time error).

So the idea is to use a quartz oscillator only once an hour, or day, or week or month. When the oscillator is operational it checks the internal operation of the pendulum clock and makes adjustment to impulse energy. But for the rest of the time, the quartz is off. This scheme implements a unique form a hybrid pendulum clock where the clock is truly free almost all the time. Quartz still serves a role, but it is not an active one. While never approaching a Shortt in terms of purity it certainly would outperform a Shortt as a timekeeper. Not only that but it could produce a nice report of corrections made over the months or years it is operational.

Summary

With ten examples of hybrid pendulum clocks I have now run out of fingers. I'm sure some ideas have been overlooked but this list serves to demonstrate that many methods do exist and their range of control varies widely. Some are nearly 100% quartz clocks while some are nearly 100% free pendulum clocks.

They all share the common feature of integrating quartz into the pendulum clock. The role of quartz varies considerably, sometimes obvious and sometimes subtle. Those making hybrid pendulum clocks should make sure they understand what role quartz has in their design.

The quality of quartz needs to be understood. Poor quality quartz will almost certainly degrade the performance of the hybrid; the performance of the hybrid is limited by the quartz. Beyond some point higher quality quartz may not increase the performance of the hybrid; the performance of the hybrid is limited by the pendulum.

Is Self-Timing or Self-Regulation Possible?

Consider the typical implementation of amplitude regulation by measuring bob velocity at the bottom of the swing with a narrow optical slot and quartz oscillator. Immediately some may object to using quartz. But we use quartz because we need a high frequency – the higher the frequency the more precisely the velocity can be measured using the flag and gated oscillator technique (100 kHz is good; 1 MHz is even better; 10 MHz is more than enough).

But what if, to avoid that objection, you were able to make the pendulum itself generate a 100 kHz signal! The beauty of this approach is that removes quartz from the equation. The pendulum would be able to make its own measurement of velocity and thus regulate its own amplitude, with no need for quartz at all. Sounds good, right?

But let's dig deeper. I can imagine some kind of sub-micron optical grating; a grating so fine that it generates pulses at 100 kHz as the bob passes over it. Using this clever technique the pendulum creates a high frequency timebase right when you need it the most.

But here's the problem. We measure velocity as distance over time. The distance is fixed (usually a small gap on the order of 1 mm). The accuracy of the velocity measurement is as accurate as the timebase. And if the timebase is coming from the pendulum itself, the "velocity" measurement number will *always be the same*, regardless of the actual velocity. That's because if the bob speeds up so does the timebase. In other words, it is not possible to use time over distance as a timebase for a distance over time measurement.

To further understand the issue, consider what the units of a measurement like 0.052 meters/second really mean. Whose meter and whose second are these units? Is it a SI second, or a quartz second, or a second as defined by the pendulum? If the former then you have an external timebase; if the latter you are *measuring time with your own time*.

And so we come to the following conclusion. Is it incorrect to say, as I casually did above, that we use quartz "because we need a high frequency". That is not why we use quartz. We use quartz because in order to make a scientific measurement we need an *independent* reference to compare against. Furthermore, for the results to have relevance the timebase needs to be *more stable* than the signal being measured.

So when you select a quartz oscillator for your pendulum clock what you are implicitly doing is using a proven highly stable rate- or time-keeper for use as a reference for some timing measurement of a pendulum timekeeper. This is why using quartz with pendulum clocks raises serious questions. You may think only of the high frequency. But in reality the *stability* of quartz and the *independence* of quartz are the key attributes. One kind of stable clock is being used to improve another kind of stable clock. There's nothing wrong with this idea of a hybrid clock, but don't confuse it with a free pendulum clock.

Conclusion

To summarize, the key points are:

- Mixing quartz oscillators with free pendulum clocks creates a hybrid clock with almost certain improved performance (that's why people do it).
- Obviously the quartz is somehow responsible for the performance improvement. This is not due to quartz accuracy per-se, or its high frequency, but rather its superb short-term stability.
- The primary motivation is amplitude control. Using quartz to finely regulate amplitude makes period that much more stable which improves overall timekeeping because it reduces circular error.
- The issue of control is not black and white: there are many levels of hybrid pendulum clocks and many ways to improve the performance by using quartz, from direct drive to occasional correction.
- There is almost no limit to the accuracy of a hybrid pendulum clock. The results should improve as more sensors, more data, more precision, and more invasive control is used.
- The complexities of the control system and the quality of the quartz oscillator have a direct effect on hybrid pendulum clock performance, for better or worse.
- One may study the degree to which quartz helps the hybrid pendulum by deliberately detuning the frequency of the oscillator or by deliberately corrupting the stability of the oscillator.
- The interaction between short-term and long-term stability is worth a paper, including the role of quartz Q and pendulum Q. The Littlemore clock is a hybrid. More on that next.

A copy of my pendulum papers is at www.LeapSecond.com/hsn2006 .

It is not my intent to take sides on what is a "proper" pendulum clock. Obviously there is an honorable charm in a fully mechanical or even electro-mechanical free pendulum clock, like those we have seen for centuries. On the other hand it is interesting to see what can be done when adds a quartz oscillator to the equation.

I want to encourage anyone working on a hybrid clock to continue to explore the potential of this interesting mix of technology. A clear understanding of quartz, measurement, and control should help remove some of the mystery and controversy surrounding hybrid pendulum clocks. I am particularly interested in efforts to explicitly quantify the role quartz plays in the success of these clocks.