A *Dream* Pendulum Clock (Tides, part 3)

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In part 1 of this article the origin, derivation and magnitude of lunar/solar tides was discussed. We know that tides cause subtle changes in pendulum clock rate, resulting in measurable instability in the very best pendulum clocks.

Part 2 explored the use of Allan deviation statistics, particularly plots and techniques which better expose the "tidal signature" of a precision pendulum clock.

Introduction

Like many of you, at one time or another I wondered if I could make the best pendulum clock. True, over the years I have come to appreciate that like music, art or architecture, there is no one *best*. Some clocks are exceedingly beautiful, others cleverly complicated, some utterly unique and masterfully constructed with unusual materials.

But one objective way to determine the best pendulum clock is of course to measure its raw timing accuracy, or stability. After all, these are timekeepers. So how hard could it really be to make the best performing pendulum clock?

There is a tendency in our computerized 21st century to think that anything done in the old mechanized 19th or 20th century could be done better today if we simply put our advanced minds, our googling fingers, and our modern technology to it. It seems in irresistible challenge.

But respect for the old masters is in order. Anyone can make a pendulum clock but it is clear that to create a world-class precision pendulum clock, a wide variety of expert skills and life experience is required; from metallurgy and fine machining, to vacuum systems and electronics, from mathematics to physics, from installation to instrumentation. The final result is only as good as the weakest link.

A race to beat the performance a Shortt free pendulum clock is fair competition, but the question is how difficult would such a challenge be and by what margin would one *win*? Even a casual reading of pendulum clock history suggests that making a new world's best pendulum clock is more than a few weeks or even months of work. It could be years. When jumping into such a life-consuming project it may be useful if not prudent to know ahead of time the potential cost and the actual reward.

If you could develop a modern pendulum clock that is two or three or ten times better than a Shortt or Littlemore, one that could rival quartz clocks, or maybe even atomic clocks, would that not be worth all the effort? I think so. But how can you know ahead of time if this goal is even possible?

To me it appeared that amateur horologists were devising new precision pendulum clocks simply guessing or hoping that they would be better than those in the distant past, but no one was looking at the limit, or perhaps even aware that there are timekeeping limits to pendulum clocks.

Expose the Limits

What if with extreme research, unlimited budget, and heroic effort an organized group of horologists solved all known problems of suspension, impulse, temperature, humidity, barometric pressure, vibration, rod creep, circular error, and escapement error? What if we used the most advanced materials known; not just aged invar but fused silica or Zerodur? What if we used the most advanced magnetic rather than mechanical impulse, optical rather than electrical detection, and the best magnetic isolation and bedrock installation, away from traffic, trains and even airplanes? Many of us have dreamed of such a clock.

The result would be an ideal free pendulum clock – affected by gravity *only*, with not a hint of mechanical or environmental flaw. But three facts remain: we are on planet Earth, pendulum clocks need gravity to operate, and very subtle changes in the acceleration of gravity occur due to the tidal effects of the dynamics of the earth-moon-sun system. There is no perfect clock.

As seen earlier the relationship between g and pendulum period is well-known. We know that astronomical computer programs exist which can compute tidal forces for any place, date, and time. It turns out they can be modified to also compute changes in g, which then predicts the deviation in pendulum period at any instant in time, and progressive time error over any duration of time. A year of data can be generated in a fraction of a second.

From a data set of progressive time errors it is simple to compute stability with Allan deviation statistics, whether that raw data comes from precise measurements of a real pendulum or whether that raw data comes from precise astronomical computer modeling.

In ***figure 3a*** we see the Allan deviation of the *Dream* pendulum clock, "designed" and "made perfectly", whose only fault, I suppose, is that it is operating on Earth.

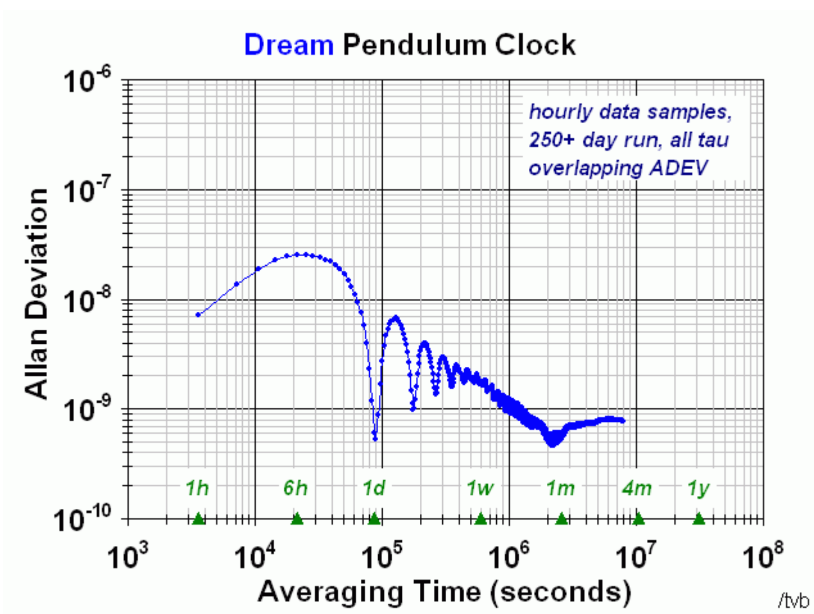

Figure 3a – Stability of best possible pendulum clock

It should not come as a surprise that the Allan deviation of this clock has many peaks and valleys and some fixed upper and lower bounds. The changes to gravity are, after all, the result of a large number of periodic effects. Pendulum period is never quite constant, and worse yet, positive and negative changes to gravity do not completely cancel over the short- or medium- or even long-term measurement. This *Dream* clock represents the fundamental limit in timekeeping performance of any free pendulum clock.

It is also clear that the Dream clock “detects tides” – because *that is all it is doing*. The best free pendulum clock is, after all, just a gravimeter by another name.

Even though it is impossible to construct a Dream pendulum clock, it is easy to imagine and its stability can be computed. The Allan deviation of this virtual clock is the timekeeping performance limit of any physical free pendulum clock.

Shortt vs. Dream

So if no clock can beat this limit, the question is how close can it get? How good is a Shortt free pendulum clock compared to this Dream clock? In ***figure 3b*** we see Shortt No. 41 and Dream plotted together. In each case the plot is created from 250 days of hourly time comparisons.

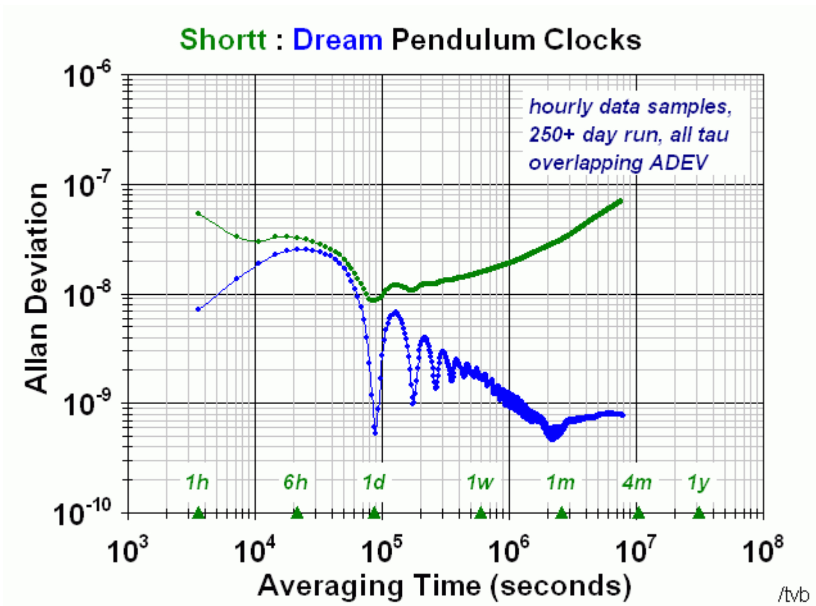

Figure 3b – Comparison of Shortt and Dream pendulum clocks

First it shows that the Shortt pendulum clock (at least No. 41) performed nearly as well as possible: for averaging times under a day it is so close to Dream that almost no room for improvement exists. Thus anyone thinking they can *improve* on Shortt in the *short-term* is advised to give up their quest.

On the other hand, when compared with Dream, it is clear just how far from optimal Shortt (at least No. 41) performs in the long-term; by a factor of almost 100. So clearly there is room for improvement here and anyone thinking they can improve on Shortt in the long-term can resume their quest, now with even greater confidence.

Later we can explore the topic of additive noise to show that Shortt is potentially better than Dream for some averaging times from about 6 to 24 hours.

Note the magnitude of tidal variation depends to some degree on latitude; the extremes of the equator and the poles are different from mid-latitudes. In the analysis here I chose 45º which is ±6º of Washington DC (Shortt No. 41) at 39º and Littlemore at 51º, as well as Seattle at 47º. Later we can investigate how latitude extremes and other gravitational variables affect the potential performance limits of pendulum clocks. Depending on the definition of a *free* pendulum clock tidal compensation can be considered as well.

You might think that creating the Dream clock in software would eliminate the need to create a real clock. But it is not the same thrill.

In fact, now that we see what the real limits are, it is all the more challenge to build a pendulum clock that meets the limit. So the challenge is not just to beat Shortt, but to create a real clock that works like a Dream.

Conclusion

Combining accurate computer modeling of lunar/solar tides with Allan deviation stability plots, we can see for the first time how good a free pendulum clock can be here on Earth. Real pendulum clock performance can be compared with theoretical performance to see how much room there is for improvement at different averaging times.

For modern pendulum clock makers this is both bad news and good news. This analysis reveals that for short-term timekeeping Shortt is already at the limit. But there is significant opportunity for exceeding the long-term performance of Shortt No. 41. Knowing what the precise limits are can help focus engineering and measurement effort where it is most needed.