of its small temperature coefficient. Many Shortt clocks have shown a progressive slowing down, as if the pendulums were slowly lengthening or as if the bobs were settling. On one of the clocks at Greenwich an invar bob was substituted in 1929.

Fused quartz has been suggested by a number of people for the pendulum rod. I know of no installation where this material has been used and tested for a sufficient length of time to show whether or not it is superior to invar. We are making experiments at this laboratory along this line.

(*d*) If a master Shortt clock is accidentally stopped by failure of the "slave" clock, broken wire, or battery failure, etc., it can easily be started again without breaking the vacuum seals. All that is necessary is to open for a second the valve which is located in the side of the case opposite the bob. The rush of air causes the bob to move slightly. If the valve is then opened slightly and closed at regular 2-second intervals in phase with the pendulum, the full amplitude can be built up in a minute or two. Then the pressure can be pumped down to the original amount.

(*e*) The brass case of the Shortt clock contains four bolt-holes for fastening the clock to the wall. Only the upper two holes should be used. The lower part of the clock should merely press against a layer of plaster of Paris placed on the wall. When four bolts are used any shifting in the masonry necessarily puts a strain on the case, causing a slow leak. In each of the clocks at Tuxedo a slow leak developed in about six months after installation, and in each case it was instantly cured when the lower nuts were loosened. I also understand that several other installations have had the same trouble.

While the installation at Tuxedo was made chiefly for experimental purposes, the recording, with some interruptions, has been continuous, and is being continued under somewhat better conditions. It has, however, been remembered that it is possible to carry out experiments which are denied to an observatory, where the clocks are needed for the time service and for astronomical observations, and it is hoped that these opportunities can be further utilised.

*Tuxedo Park, N.Y.:*
1931 *February* 16.

*Analysis of Records made on the Loomis Chronograph by Three Shortt Clocks and a Crystal Oscillator.* By Ernest W. Brown and Dirk Brouwer.

The chronograph, clocks, and oscillator have been described by Mr. A. L. Loomis in the previous paper. In this paper we give the results of a somewhat extensive analysis of records furnished by Mr. Loomis, extending over several months.

*Frequency of Reading.*—Let us denote the time shown by the quartz crystal system by Q and those by the three Shortt clocks by $C_1$, $C_2$, $C_3$. Then if $0^s{\cdot}001$ be named a milli-second (ms.), the record consists of readings every $30^s$ of the differences

$$C_1 - Q, \qquad C_2 - Q, \qquad C_3 - Q,$$

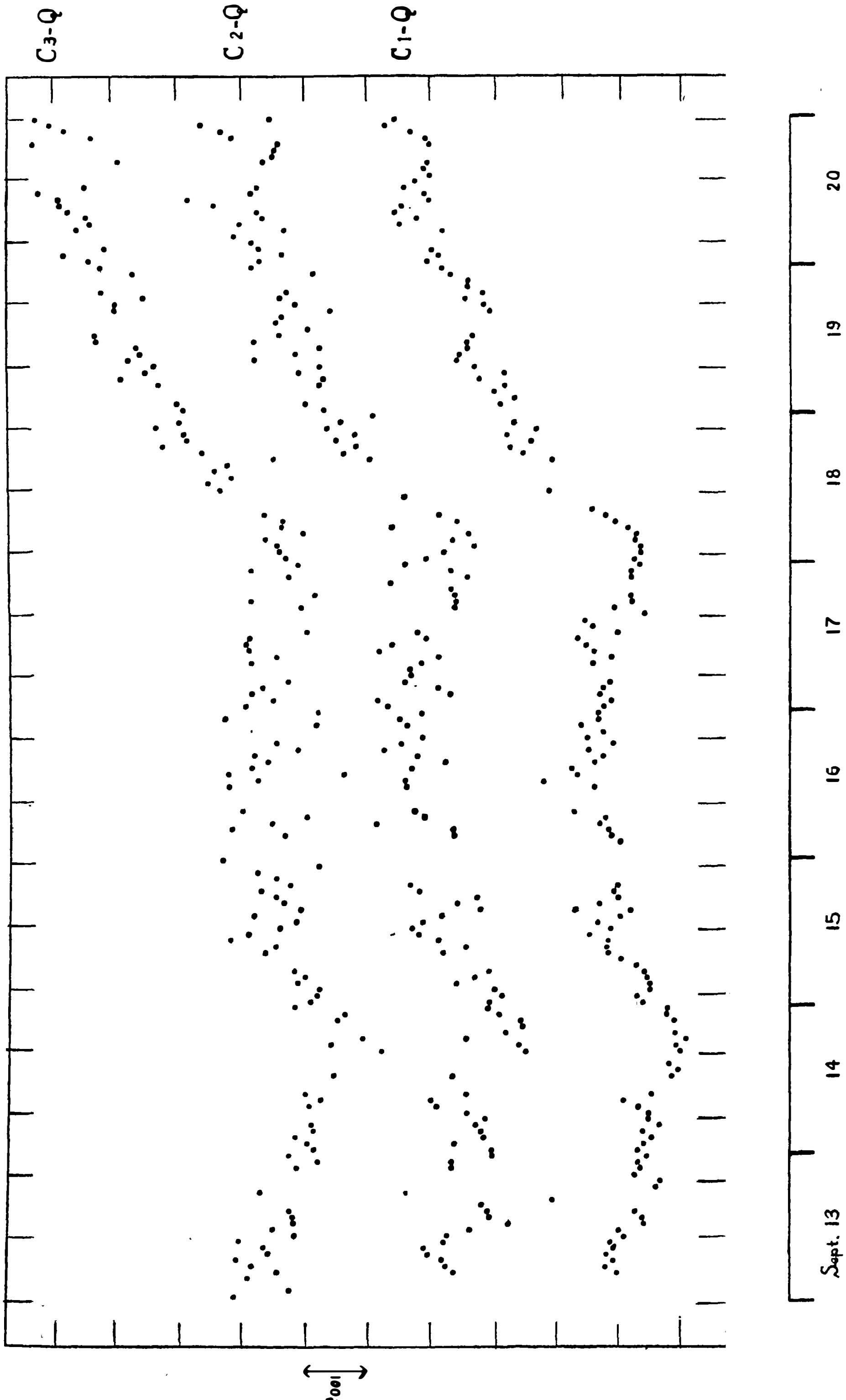
to the nearest ms. The three clocks thus give 360 records per hour or 8640 per day.

Several problems were presented. The first was to discover the degree of constancy of the rates of the crystal and clock and chronograph systems over intervals of the order of a minute, an hour, a day, a month. . . . In this connection it has to be remembered that between the pendulum of each clock and the Loomis chronograph there is a chain of mechanical and electrical links, and that each of these links could cause small random errors additional to the errors peculiar to the crystal and the pendulums. The nature of these recording mechanisms and of their connections was, however, such that they should not have been able to produce accumulated errors at any time greater than a few ms. at most.

Examination of the records showed that changes greater than 1 ms. during any $30^s$ interval were rarely present. Thus the perforations which gave the record for any one clock deviated from a straight line drawn to follow their mean direction by amounts rarely greater than 1 ms. over intervals of 20 minutes or so. In fact, the irregular changes in the hourly rates as obtained from reading and averaging the results from all the perforations were generally considerably less than 1 ms. Fig. 1 gives a sample of these hourly rates, covering 8 days.

Economy of effort requires that the frequency of reading of the records shall not be greater than that demanded by the problem under consideration. It was therefore necessary to find out what was the loss of accuracy caused by taking readings at longer intervals than the $30^s$ given on the record. For this purpose, for the days 0–54 (1929 September 7, $0^h$, to October 31), every record on the sheets was read and averaged into hourly means. This material is denoted below by the symbol M. As stated above, it was evident that oscillations greater than 1 ms. during any hour were rarely present, so that these hourly means should have a probable error not greatly in excess of 1/11 that of a single reading.

*Dispersion of the Hourly Means.*—Three samples of two and one-half days each were taken, and smooth curves were drawn through the points representing the hourly rates for clocks minus crystal and for the differences between the clocks. If $\mu_0$, $\mu_1$, $\mu_2$, $\mu_3$ be the mean errors of the readings of the four mechanisms against an ideal clock with no dispersion and assumed to be represented by the smooth curves, we obtain six equations giving the sums of values of the $\mu^2$ in pairs. The deduced values were consistent to less than 5 per cent. and gave $\mu_0 = \pm\, 0^s{\cdot}00013$, $\mu_1 = \pm 0^s{\cdot}00014$, $\mu_2 = \pm\, 0^s{\cdot}00023$, $\mu_3 = \pm\, 0^s{\cdot}00024$. The probable errors of the hourly means for the material M are therefore in the neighbourhood of $\pm\, 0^s{\cdot}0002$. Only a portion of each error is due to the fact that the individual reading is made to the




Hourly rates: Clocks minus Crystal.

Fig. 1.

nearest ms., since the probable error due to this cause should be less than $\pm 0^s{\cdot}0001$.

Owing to the method adopted, the irregularities considered here should diminish in taking means according to the Gaussian law. The comparisons of the results obtained below for the lunar term indicate that this is the actual fact.

In the sample of the record shown in fig. 1 it is noticeable that the dispersion for Clock 1 is considerably less than that for either of the other clocks, and this is the case throughout the whole interval of 55 days used to find the material M. These dispersions are probably due mainly to variations in the signalling systems between the pendulums and the chronograph. Thus the smaller dispersion of hourly rates shown by Clock 1 indicates only that its signalling system is more regular than those of the other clocks and not that it is a better time-keeper. In fact, the indications given below show it was more subject to sudden changes of rate, which were maintained over intervals of many days.

Next, the material for days 0–146 (1929 September 7 to 1930 January 31) was read at hourly intervals only by noting the position of the intersection of the line of perforations with that giving the hour. As the angle of intersection of these two lines was greater than 70°, the "hour" line was needed only to an accuracy of a minute or so; actually it is obtained to within 5 or 10 seconds. The probable error of such a reading, denoted by H, should be less than 1 ms., and a comparison of readings with the results of the material M showed that this was the case.

*General Treatment of the Records.*—As stated above, the material M consists of the hourly means of the readings taken every $30^s$ for days 0–54, and the material H consists of the readings at $0^h$, $1^h$, . . . for days 0–146. The times are E.S.T. = G.C.T. $- 5^h$.

Constant rates were subtracted from the readings of $C_2 - Q$, $C_3 - Q$, so as to make the average rates of these as near $C_1 - Q$ as possible.* These were determined from the material M but served sufficiently for the period of 147 days. The object was merely to diminish the magnitudes of the numbers. The amounts applied were, per day,

$$0^s{\cdot}048 \text{ to Cl. } 2, \qquad 0^s{\cdot}216 \text{ to Cl. } 3.$$

The hourly differences were then formed for both M and H. At this stage the gaps in the record were bridged by the method given below, for the convenience of having a continuous record. From this material the differences $C_1 - C_2$, $C_1 - C_3$ (and $C_2 - C_3$ as a check) were formed, and thereafter the results for $C_1 - Q$, $C_1 - C_2$, $C_1 - C_3$ were used. The procedure of using the hourly differences, giving small numbers, can be used for all operations which involve only additions and subtractions. At any stage the actual values of the readings can always be found. The final stage before a harmonic analysis was the recovery of these readings.

* The notations $C_1 - C_2$, etc., have the opposite signs to those given by the clock readings. They were used for convenience in the analysis.

*Gaps in the Record.*—Owing to interruptions in the telephone line connecting the oscillating crystal (in New York) and the chronograph (in Tuxedo), the latter stopped on a number of occasions and remained at rest until the stoppage was discovered and the chronograph started again. These stoppages in no way affected the running of the clocks or the oscillations of the crystal, nor did they affect the determination of the rates between clocks and crystal while the chronograph was running. The readings were only lost during the stoppages.

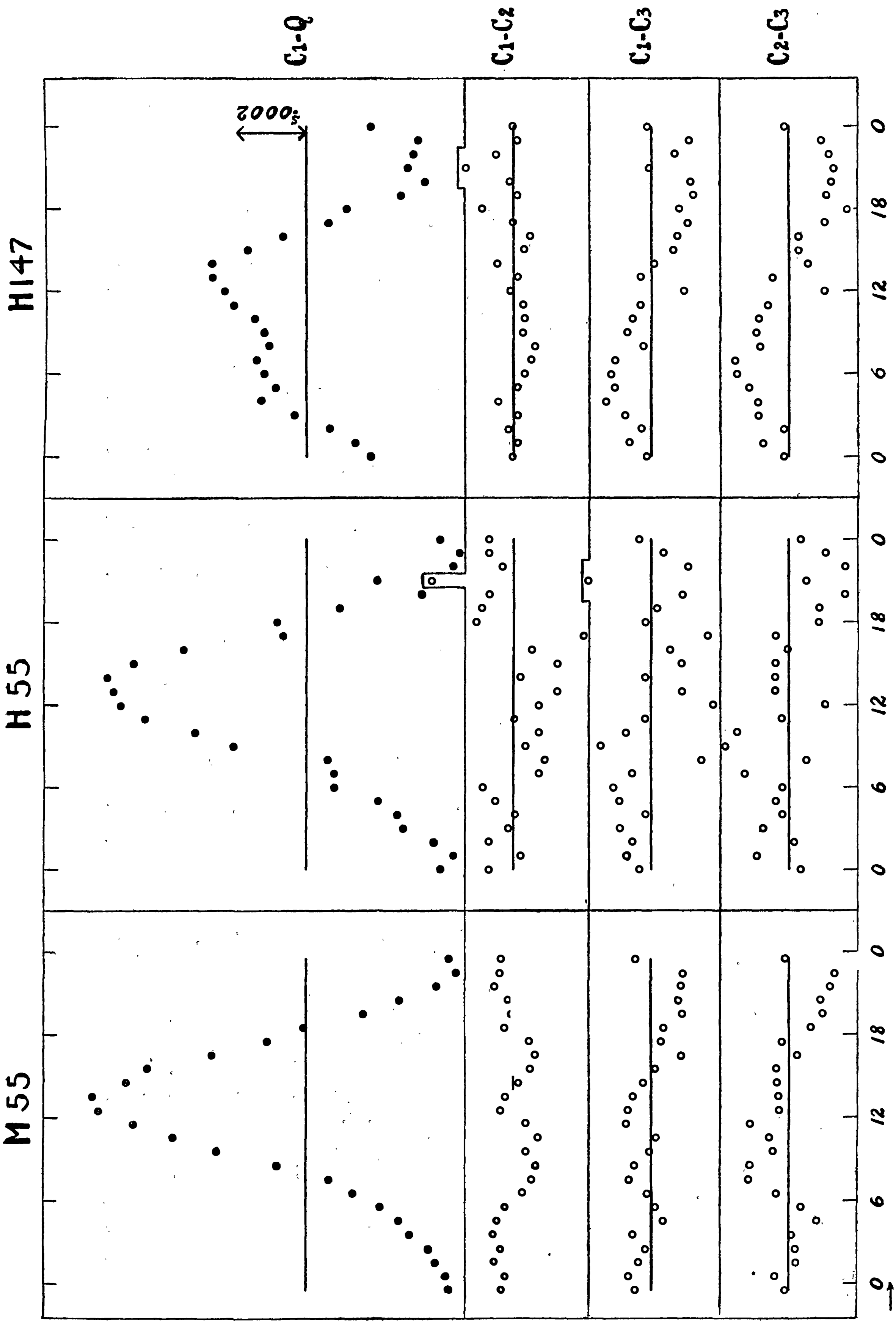
To avoid the inconvenience of analysis with discontinuous series, the gaps were bridged by making use of the fact that the rates of the clocks and crystal were nearly constant over periods of several hours. By finding the rates just before and just after the break, and assuming that they changed uniformly in the interval, the gaps could be filled without errors greater than 1 or 2 ms. in the clock readings. The readings of the clock differences $C_1 - C_2$, $C_1 - C_3$, $C_2 - C_3$ are not affected, since for them only multiples of 120 ms.* had to be found and there was never any doubt as to these. For clock-crystal only one number, which is equivalent to the change of the zero of the record, has to be found, and as it could be found separately from each of the clocks, there was little doubt as to its amount. In the interval, September 7 to October 31, there are seven gaps of between 15 and 24 hours, and a few additional ones of shorter interval. The bridging of the gaps by constant rates diminished the probable error of any systematic change, but the total extent of the gaps was not great enough to do so materially.

*Analysis for Oscillations with Periods of the Order of Half a Day to a Day.*—As soon as the hourly rates obtained from the material M were plotted, it was seen at once that there was a daily oscillation in these rates with a half amplitude of ·2 or ·3 ms., and that this oscillation was substantially the same for all three clocks. It was noticed, however, that these daily curves showed considerable differences in shape, amplitude, and phase from day to day. In over 50 per cent. of the cases where a minimum could be detected graphically, this minimum occurred near $17^h$ or at 5 p.m. An analysis for a daily period was made in the following manner.

The mean values of the rates for each hour of the day were obtained for $C_1 - Q$, $C_1 - C_2$, $C_1 - C_3$ with the material M, $0^d$–$54^d$; H, $0^d$–$54^d$; H, $0^d$–$146^d$. These rates have progressive increases or decreases due to the secular and long-period changes. Constant changes of rates were first applied to each series in order to free them from these changes. The accumulated errors at each hour of the day were then obtained, and these showed the average errors of the clocks at each hour of the day. The results showing these errors in the readings of $C_1 - Q$, $C_1 - C_2$, $C_1 - C_3$, $C_2 - C_3$ are plotted in the twelve curves of fig. 2.

It is evident that there is little, if any, trace of the effect in the clock differences. Clocks 1, 2 appear to be free from it; there is a slight trace in Clock 3. As the temperature controls of the three free pendulums and of the crystal are very close, the source of the effect is probably to

* The chronograph used during that time was one with 120 teeth instead of the 100 teeth in the later type described by Mr. Loomis.


M 55
H 55
H147
C1-Q
C1-C2
C1-C3
C2-C3
0
6
12
18
0
6
12
18
0
6
12
18
0

be found in temperature changes in the signalling system, including the forty-mile telephone line from New York. The average hours of change of rate, namely, 10 a.m. and 5 p.m., are significant. The irregular character of this diurnal oscillation is also indicated by the fact that its average amplitude over a period of 147 days is considerably less than that over a period of 55 days.

Another point which is evident from the figure is the evidence that the material M gives smoother curves than the material H. This might be expected in view of the relation of the amplitude to the accuracy of measurement.

*The Influence of the Moon.*—If the daily oscillation just discussed was due to some portion of the recording mechanisms, as seems probable, and was not present in the oscillations of the crystal or of the free pendulums, some other test of the accuracy of the latter over periods of the order of a day or half a day is needed. For this purpose the lunar day, the interval between successive transits of the moon—which on the average is $24^h\ 50^m$—was selected. If averages over intervals of multiples of 29 days be taken, any effects due to influences with periods of a solar day or of its submultiples should be eliminated. A preliminary calculation showed that the direct attraction of the moon on the pendulum should give an oscillation with a half amplitude of $0^s{\cdot}00015$ for the period of half a lunar day.

It was of interest to see whether so small an effect could be detected.

*Analysis for the Lunar Semi-diurnal Term.*—The material used for the analysis of the term with the period of a solar day was adapted to the lunar day in the manner devised by G. H. Darwin for tidal analysis,* with the particular form given to it by one of us.† In this form, the portions due to secular and long-period changes, which had to be subtracted in the case of the solar day, automatically disappear. If averages over multiples of 29 days be used, there should be little or no sensible effect of the somewhat irregular solar day term. In this method the analysis is made by finding the nearest solar hour at which each lunar day begins, and putting under that hour the corresponding values of the function. If for any particular solar day the value chosen is that at $h$ hour, the remaining twenty-three values at intervals of a solar hour are taken to be the values at $h+1, h+2, \ldots 23, 0, 1, \ldots h-1$ hours. This is done for each solar day, and twenty-four averages are then obtained. These are the averages at $0^h, 1^h \ldots 23^h$ from the beginning of the lunar period, but it is sufficiently accurate to consider them as proceeding at intervals of 1/25 of the lunar period, with the last missing. For the degree of accuracy needed here, each can be taken in the Fourier analysis as separated by 1/24 of the period.

The analysis was made

(*a*) with the material M for days 0–54,
(*b*) with the material H for days 0–54,
(*c*) with the material H for days 0–146,

and each was carried out for $C_1 - Q$, $C_2 - C_1$, $C_3 - C_1$.

* *Coll. Works*, vol. i. p. 216. † E. W. Brown, *Amer. Jn. Sci.*, Series 4, **39**, 386, 1915.

The results are shown in the twelve curves of fig. 3. As might be expected, the differences between the clocks show no traces of the term, for the effect, being a change in the value of $g$, should be the same on all three clocks, and the oscillation of the crystal should be independent of small changes in $g$. The smooth curve under M55 for $C_1 - Q$, obtained with the material M, may be compared with that under H55 for the material H, and with that under H147 for the material H over an interval three times as long. There is evidently a lunar daily as well as a lunar half-daily term present: this is smaller in H147 than in M55 or H55.

The Fourier analysis for a term with the period of half the lunar day gives

| | | | | |
|---|---|---|---|---|
| | d d | s | | h m |
| (M), $C_1 - Q$, | 0–54 | + 0·000128 | sin 2($\phi$ − | 0 0) |
| (M), $C_2 - Q$, | ,, | 147 | | + 0 13 |
| (M), $C_3 - Q$, | ,, | 106 | | − 0 36 |
| (H), $C_1 - Q$, | 0–146 | 150 | | − 0 24 |
| (H), $C_2 - Q$, | ,, | 135 | | − 0 25 |
| (H), $C_3 - Q$, | ,, | 127 | | − 1 0 |
| Theory, direct attraction of moon, | | 153 | sin 2$\phi$ | |

As the probable errors of the coefficients deduced from (H) lie between $\pm\ 0^s{\cdot}00001$ and $\pm\ 0^s{\cdot}00002$, the agreement between them is satisfactory. The method of procedure has diminished the coefficients by perhaps 5 per cent.

The same effect would be produced if the clocks were given a vertical oscillation with a semi-amplitude of 2·5 inches and with the same period. An amplitude of 1 inch could be detected in a month's observations. This fact is remarkable testimony not only to the sensitiveness of the Shortt clocks, but also to that of the crystal oscillator and of the Loomis chronograph.

*Theory of the Lunar Effect.*—The calculation of the direct attraction of the moon on the pendulum is obtained from the tidal potential V. The change $g_M$ in $g$, due to this potential, is given by

$$-\frac{g_M}{g} = \frac{2M}{E}\sin^3\pi(\tfrac{3}{2}\cos^2 z - \tfrac{1}{2}),$$

where M, E are the masses of the earth and moon, $\pi$ is the parallax of the moon, and $z$ its zenith distance at any time.

The coefficients of the various periodic terms in the expansion of $\cos^2 z$ can be obtained from the development by G. H. Darwin.* For the principal semi-diurnal lunar tide he gives a coefficient ·454, so that with $E = 81{\cdot}5M$, $\sin^3\pi = 1/219000$, we obtain

$$-\frac{g_M}{g} = 2\frac{\cdot 454}{1{\cdot}19\times 10^7}\cos^2\lambda\cos 2\phi,$$

* *Coll. Works,* vol. i. p. 6.

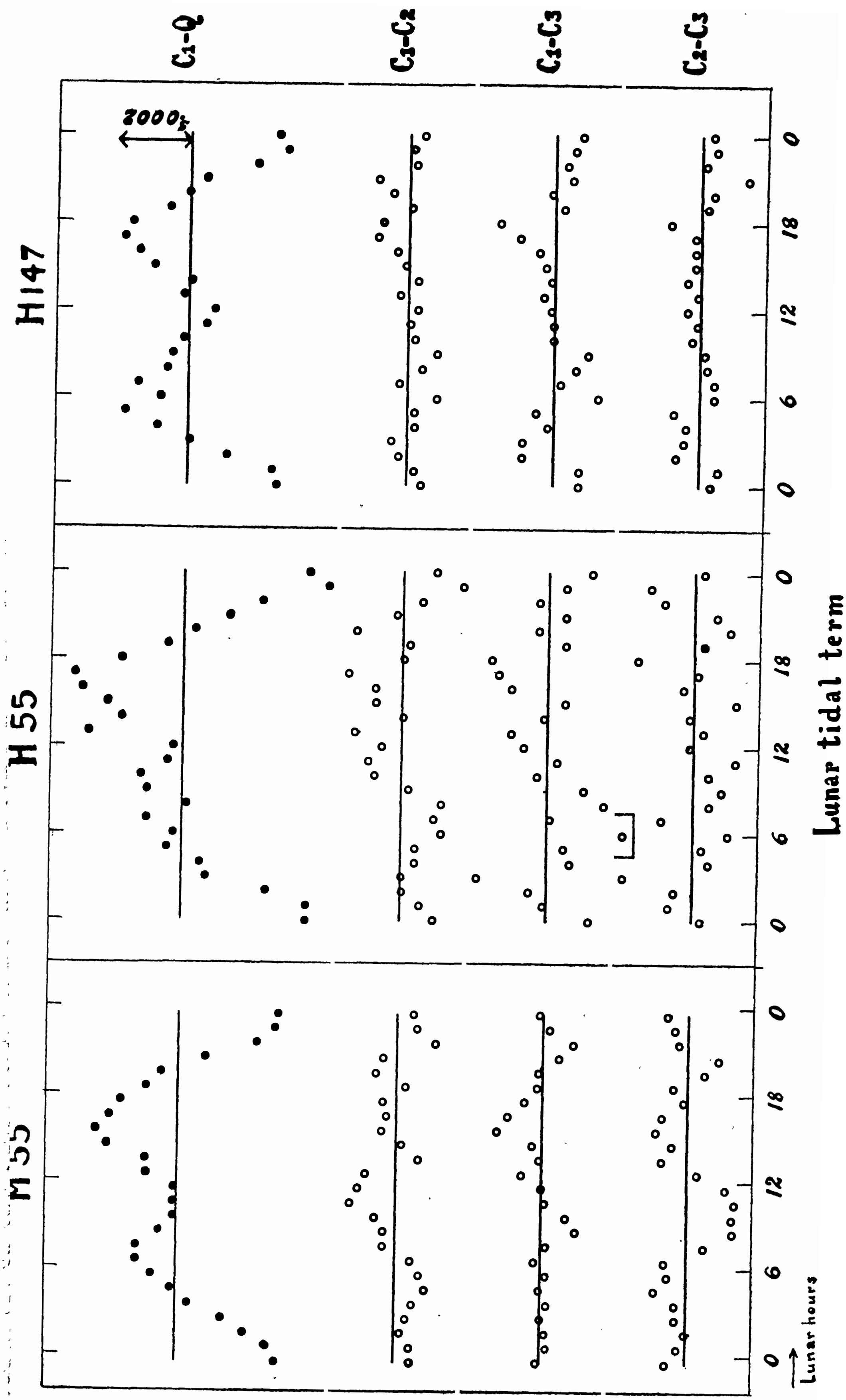

H147
H55
M55
$C_1$-Q
$C_1$-$C_2$
$C_1$-$C_3$
$C_2$-$C_3$
2000
0
6
12
18
0
Lunar hours
Lunar tidal term

FIG. 3.

40

where $\lambda$ is the latitude of the place of observation, and $2\phi$ is the argument of the tide having a period of $12^h\ 25^m$. With $\lambda = 41^\circ$ (the latitude of Tuxedo Park) we obtain

$$\frac{g_M}{g} = -4\cdot3 \times 10^{-8} \cos 2\phi.$$

Hence, if P be the half-period of the pendulum ($1^s$) and $\delta$P the change in P due to $g_M$, we have for the error of the clock at any time,

$$-\int \delta P dt = \tfrac{1}{2}\int \frac{g_M}{g} dt = -2\cdot15 \times 10^{-8} \times \frac{44714^s}{2\pi} \sin 2\phi = -0^s\cdot000153 \sin 2\phi,$$

where $\phi$ is zero when the moon is on the meridian.

This direct attraction is, however, only a portion of the change of $g$ caused by the moon. First, there is the change of height caused by the bodily earth tide due to the attraction of the moon: denote the ratio of the effect of this to that of the direct attraction by $h$. Secondly, there is the effect due to the redistribution of mass caused by this bodily tide, giving a ratio $k$. These have the same periods and phases as the terms due to the direct attraction, and both of them are positive. According to Jeffreys *

$$h = \cdot60, \qquad k = \cdot27,$$

so that these two alone would increase the coefficient by nearly 90 per cent.

There are, in addition, two "local" effects. One is the depression of the ocean-bed and near-by coast caused by the accumulation of water at high tide. Darwin estimates † this at $-(h+k)/7$ for the Atlantic coast. Another local effect is that produced by the change of attraction of the ocean tide. If this were uniform all over the earth, it would have the same sign as $k$—that is, the component of the attraction in quadrature would be greater than that when the moon is on the meridian. In the northern hemisphere the portion of the quadrature component contributed by the Atlantic is small; that by the North Pacific Ocean will be larger. The chief effect will be contributed by the portion in the southern hemisphere where the motion of the tide is not much disturbed by the presence of land masses, so that the net effect is to give a negative coefficient. If we accept the results from the Shortt clocks as showing the total effect, it would appear that these local effects balance $h+k$. An attempt to calculate them is therefore desirable. The discussion of these points is pertinent to the subject of this paper, however, only in so far as it has a bearing on the degree of accuracy which is obtainable from the clocks and chronograph.

The tidal theory also gives terms which have nearly, but not quite, twice the period of the semi-diurnal tide discussed above. Darwin gives coefficients $\cdot19 \sin 1\lambda$, $\cdot18 \sin 2\lambda$, $\cdot08 \sin 3\lambda$ for three of them as compared with $\cdot454 \cos^2 \lambda$ for the principal semi-diurnal tide. The

* *The Earth*, 2nd edition, p. 237.

† *Coll. Works*, vol. i. p. 457.

first two of these give coefficients in $g$ nearly double that of the principal tide, but owing to the differences of period they will tend to disappear in averages over a long interval of analysis for the latter. There is evidently a lunar-diurnal effect in the first group of curves of fig. 3, which is probably a residual from these tides. It is noticeable that the effect is smaller for the interval of 147 days than for that of 55 days, a result which might be expected as due to the better elimination in the longer interval.

*Irregular and Secular Changes.*—Next in order should come the discussion of oscillatory changes taking place in periods of a week or more, but the existence and explanation of a marked oscillation with a period of 10·5 days and with a considerable amplitude required a knowledge of the secular and irregular changes of rate, so that these will be considered next.

The actual changes of rate under discussion are very much larger than those previously considered, and it was consequently possible to neglect the small oscillatory changes of rate taking place during a single day. The values of the accumulated errors at $0^h$ (midnight) of each day were taken from the material H, and from these were formed the daily rates, 147 values of the latter being thus available. Later, material carrying the daily rates up to $237^d$ was added. In this latter portion there were considerable gaps of some days at a time, so that it was only possible to obtain the differences between the clock rates. For these the gaps could be bridged in the same manner as before. We have therefore the daily rates for Cl − Q to $147^d$ and Cl − Cl to $237^d$.

For the investigation of these, the following device was adopted. Let any value at $t$ days be represented by $f(t)$ and put

$$f(t) = a_0 + a_1 t + a_2 t^2 + \ldots + \Sigma A \sin(nt + a) + \psi(t) \quad . \qquad (1)$$

Here the coefficients $a_2, a_3 \ldots$ represent changes of rate which in general can be confined to the coefficient $a_2$. The function $\psi(t)$ represents irregular changes. If we form the functions

$$f_1(t) = f(t+k) - f(t), \quad f_2(t) = f(t+2k) - 2f(t+k) + f(t) = f_1(t+k) - f_1(t),$$

and plot the curves with $t$ as abscissa and $f_1$ or $f_2$ as ordinates, the average rate $a_1 k$ becomes the average ordinate in $f_1$ and disappears from $f_2$. The secular change $2a_2 k$ becomes the slope of $f_1$ and the ordinate of $f_2$. The periods of the harmonic terms are unchanged, but their amplitudes and phases are respectively altered to

$$2A \sin \tfrac{1}{2}nk, \qquad a + \tfrac{1}{2}nk + \frac{\pi}{2}; \qquad 4A \sin^2 \tfrac{1}{2}nk, \qquad a + nk + \pi.$$

Some care is needed in the interpretation of $\psi_1, \psi_2$. A sudden permanent change of rate, for example, gives a change of the ordinate of $f_2$ with a single oscillation of period $2\pi/k$ at the change. An irregular oscillatory change will give one or two oscillations with a period depending on $k$ and on the interval of the oscillation.

If we put $k = 2\pi/n$, the corresponding harmonic term will disappear from $f_1$, $f_2$. If, however, the phase and/or amplitude A is not quite constant, the term may still be present with a small amplitude, and, as just stated, an oscillation with this period may from time to time appear from $\psi$. If, then, we give $k$ a value nearly equal to $2\pi/n$, and if the irregularities are not large, we can eliminate the periodic term without losing the secular changes of rate.

If $k = \pi/n$, the amplitude of the term with period $2\pi/n$ will be multiplied by four and the amplitudes of other periodic terms will be relatively diminished. The secular change $a_2$ has only half the effect in $f_1$ and one-quarter that in $f_2$, while $\psi_2$ may give an occasional oscillation of period $2\pi/n$ and thus alter the apparent phase and amplitude of the harmonic term with this period. These occasional changes, however, will have little or no effect on a harmonic analysis to find A over an interval many times that of the period.

To obtain the secular and irregular changes free from the marked 10·5-day oscillation, the value $k = 11^d$, near enough to the actual period to eliminate the term from the diagram, was taken for $k$. This requires simply the formation of the differences of the readings 11 days apart. For the reason stated earlier and because the irregular changes of rate of the crystal were considerable, the discussion is mainly confined to the differences between the clocks. The lower three curves in fig. 4 give the three plots for $f_1$; the lowest curve is, of course, the difference between the other two. The ordinates show the daily rates.

By a comparison of these curves, it is possible to detect the sudden changes of rate of each clock, on the assumption that two clocks will not have the same change at the same time. Such an inspection gives five considerable changes to Clock 1, three to Clock 3, and one doubtful one to Clock 2, in the interval of $237^d$.

The curves show at once the extent to which the clocks may be trusted over any given interval. The maximum change of rate per day is $0^s{\cdot}003 \times \frac{1}{2}$, but the maximum accumulated error of a clock reading in a week is $0^s{\cdot}05$. The main point is, however, that the continued comparison of three clocks by means of the Loomis chronograph gives a standard of time measurement over long periods because of the possibility of detecting the changes of rate of each individual clock from the differences. Four such clocks, under proper conditions of temperature and pressure control, could possibly furnish a control on the annual changes in the rate of rotation of the earth.

*The 10·5-Day Oscillation.*—Quite early in his comparison Mr. Loomis had noticed an oscillation with this period and with an amplitude of one or two hundredths of a second, and had connected it with the "beat" period of Clocks 2 and 3. It was difficult to see how the two free pendulums could sensibly influence one another, and it therefore seemed worth while to analyse the records in some detail with reference to such an effect.

In order to bring out the oscillation freed from the greater part of the secular changes, the value $k = 5^d$ was used in the formula and the resulting curves for $f_2$ were plotted. Those for the three clock differences as far as $227^d$ are shown in the upper three curves of fig. 4, and those for clock − crystal in fig. 5.

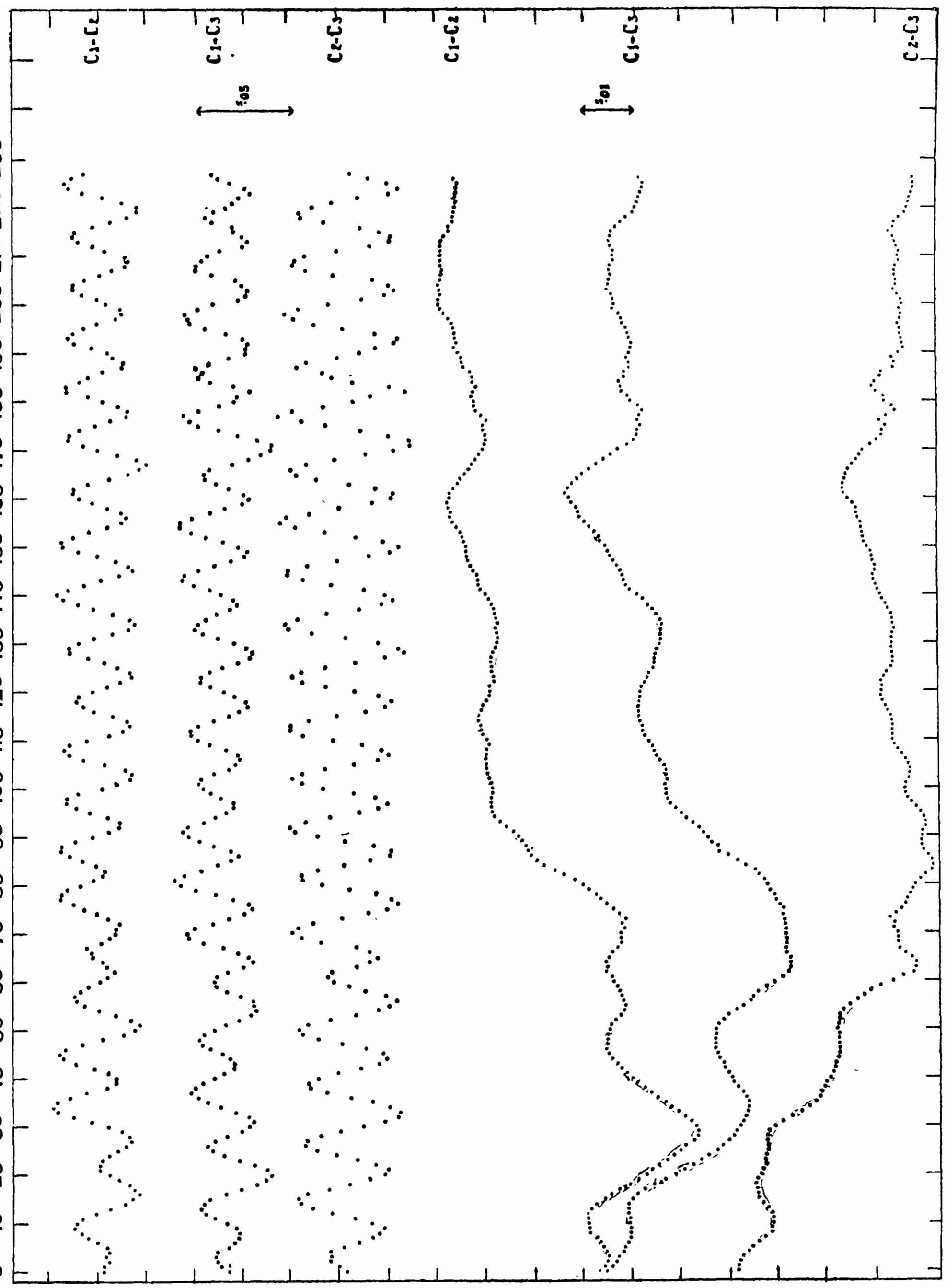



FIG. 4.—Upper Curves: The 10·6-day term in the clock *readings*. Lower Curves: Mean *daily rates*, after elimination of the 10·6-day term.

The remarkable regularity of these curves in fig. 4 can leave no doubt as to the real existence of the effect. The comparison gives a period of $10^d{\cdot}5 \pm {\cdot}1$ and a phase difference between Cl 1 − Cl 2, Cl 1 − Cl 3 of 3 or 4 days. The differences Cl − Q are much larger and show the effect only in a partial way.

The first question to settle is whether the period does, in fact, correspond to the "beat" period of Clocks 2, 3. To examine this point, we refer to the lowest curve in fig. 4, which gives the relative rates of the two clocks freed from the term. Between $0^d$ and $80^d$ this rate changes from $-0^s{\cdot}160$ to $-0^s{\cdot}196$, giving a beat period ranging from $12^d{\cdot}5$ to $10^d{\cdot}2$. Between $90^d$ and $227^d$ it changes much less and has an average value of about $10^d{\cdot}5$. In the third curve of fig. 4 it is evident

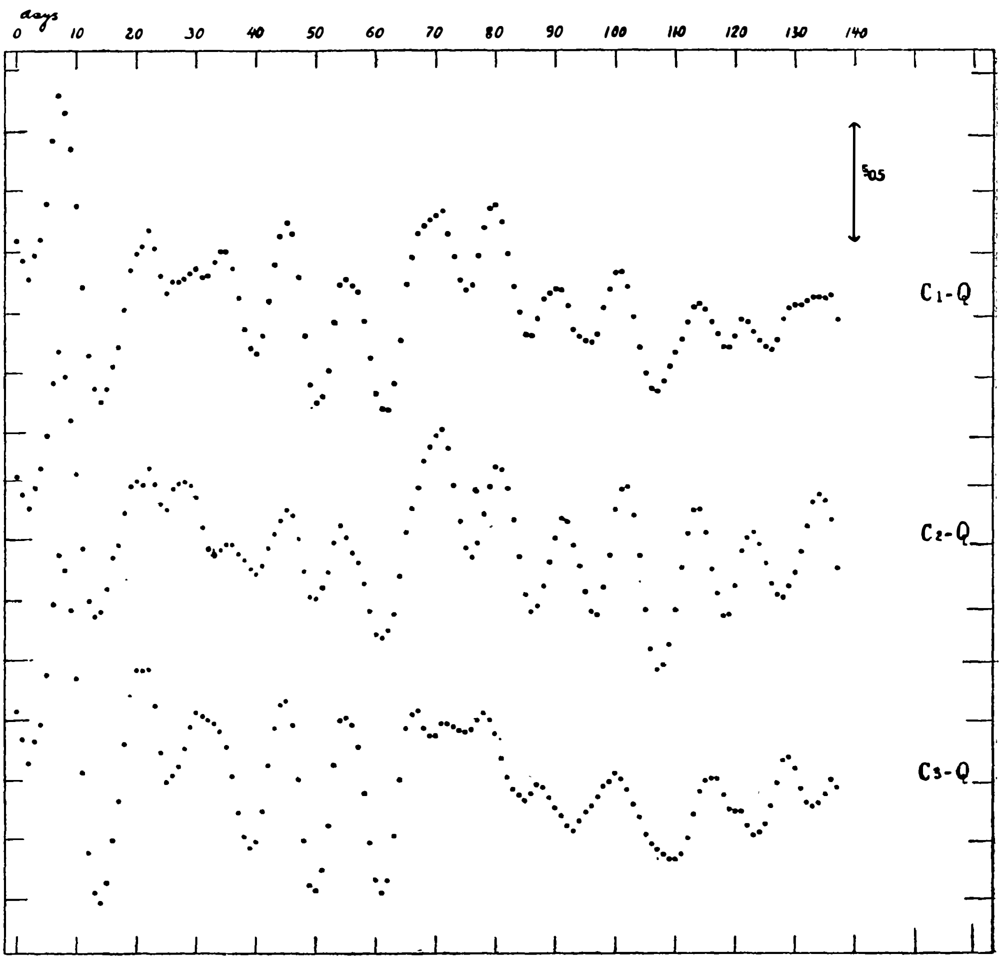



FIG. 5.—The 10·5-day term in the clock readings; Clocks minus Crystal.

that the first five oscillations have a mean period of about 11·5 days, while after $90^d$ it is 10·5 days with very little variation. The irregularity at $60^d$ corresponds to the sudden change of rate at that time. To secure more detailed information, numerical analysis must be used. This analysis has been confined to the interval between $90^d$ and $227^d$ when the relative rates were nearly constant.*

We wished to find the average amplitude and phase, particularly

* Mr. Loomis informs us that in November 1930, after the completion of this analysis, the rate of Clock 3 was purposely changed in connection with a new series of experiments, and that with the change the 10·5-day period disappeared, as might have been expected.

the latter, on account of its bearing on hypotheses as to the source of the effect. The ordinary Fourier analysis was modified in the following manner, in order to avoid the influence of variations in the length of the period. The beginning of each period was chosen at the moment when the difference of the readings of Clocks 2, 3 was an integral multiple of $2^s$, the period of oscillation of the pendulums. The daily readings during each such period were to be interpolated, so that each entry corresponded to 1/12 of the interval: thus there were twelve entries for each period corresponding to phase differences of 30°. Actually, instead of the interpolated values, those nearest to the given times were chosen, as in tidal analysis. Twelve complete periods were available, and the means for each 30° of phase were formed and analysed for a constant and a periodic term in the usual manner.

The results on the clock readings are as follows :—

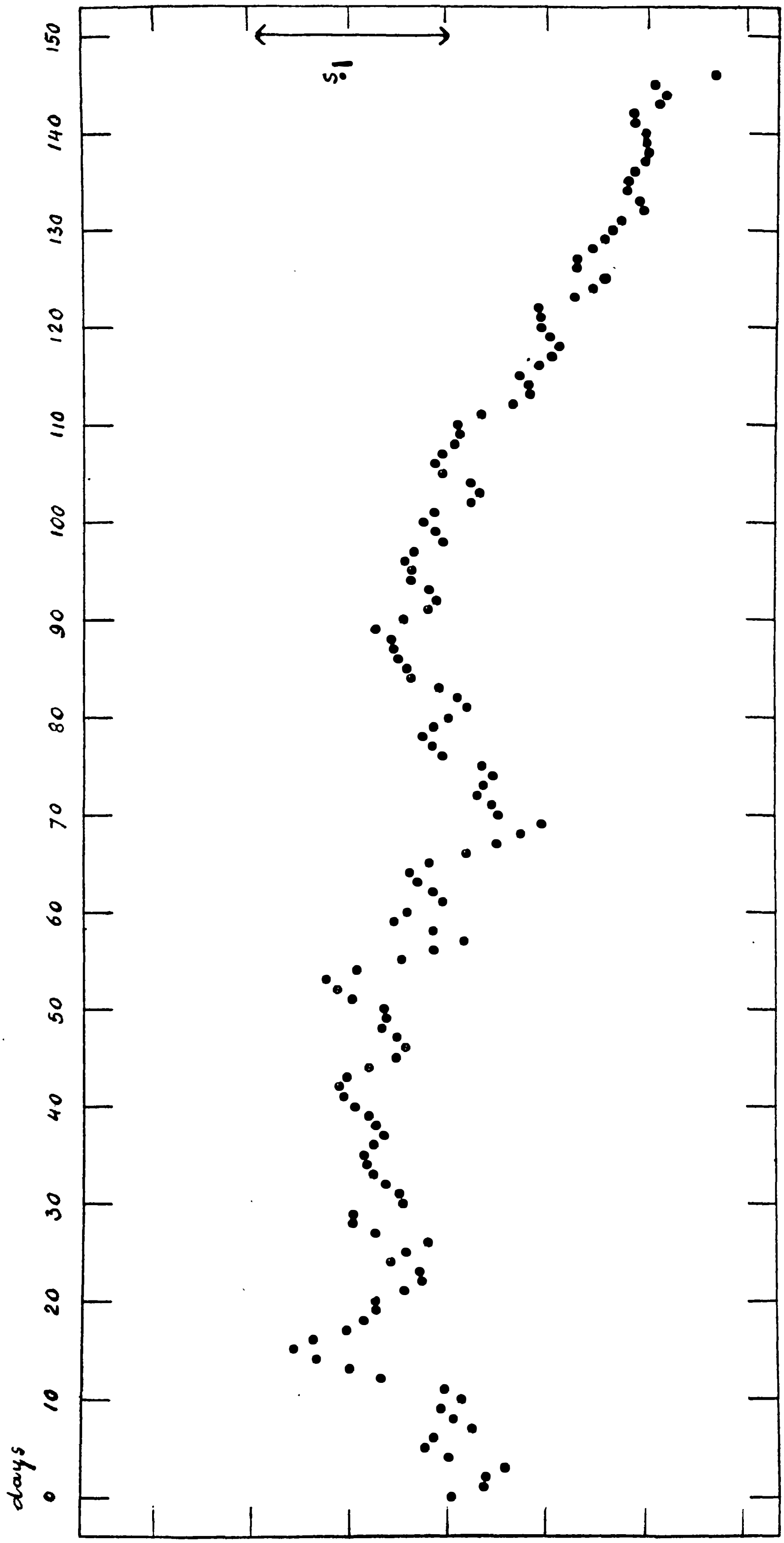
$$\text{Cl } 2 - \text{Cl } 3, + 0^s{\cdot}0271 \sin\phi\,;$$
$$\text{Cl } 1 - \text{Cl } 3, - 0^s{\cdot}0150 \sin\phi + 0^s{\cdot}0059 \cos\phi\,;$$
$$\text{Cl } 1 - \text{Cl } 2, + 0^s{\cdot}0121 \sin\phi + 0^s{\cdot}0059 \cos\phi\,;$$

where $\phi$ has the period of the "beat" and is evidently taken to be zero when the term in Cl 2 – Cl 3 is zero. The probable errors of these coefficients, as deduced from the daily rates, are less than $0^s{\cdot}002$.

According to almost any hypothesis as to the interaction of the clocks, it is difficult to see how (1) can be affected by the beat period of (2), (3). The average beat period of (1), (3) is $9^d{\cdot}6$, and that between (1), (2) is of the order of $100^d$. The latter cannot be detected owing to the irregular changes of rate, and we have been unable to find any reliable evidence of the former—if there is such a term, its coefficient is not much greater than $0^s{\cdot}001$. If, then, (1) is free from the term, the second of the results gives the effect of (2) on (3), and the third the effect of (3) on (2).

The amplitudes of the two terms are not very different, but there is a marked phase difference of about 130° which is noticeable as existing throughout the interval in the plots. Further, when the times at which the two pendulums are in the same phase are compared with the times of maxima and minima of the term in Cl 2 – Cl 3, it is at once seen that the two do not correspond, but differ throughout by 1 or 2 days. Thus the beat effect is not at a maximum when the pendulums are in the same or opposite phases.

Let us return to the possible existence of the effect in Cl 1 – Q. The first of the curves of fig. 5 appears to give evidence of its existence. When it is examined more closely, however, it is seen that between $35^d$ and $65^d$ there are three such oscillations and then a sudden reversal of phase. This is followed by four more oscillations, a change of phase, and a fading out. Now it has been pointed out that in the process of forming $f_2$ a single oscillation with a period of $10^d$ will be produced by a sudden change of rate, but it is difficult to account for four successive oscillations as due to irregular changes of rate. Nevertheless, we know that the term exists in (2) and (3), and the regularity has disappeared




Daily rates; Clock 1 minus Crystal.

Fig. 6.

from the curve for Cl 3 − Q, showing that the term in (3) is combined with an irregularity in Q of the same order of magnitude. We can only conclude that the irregularities in the crystal system are such that no reliable evidence as to the 10·5-day term can be deduced from the curves of fig. 5, and that we must rely on those of fig. 4.

In order to get rid of the introduction of spurious $10\frac{1}{2}$-day periods by the process of forming $f_2$, the daily rates of Cl 1 − Q are plotted in fig. 6. There is little, if any, trace of the 10·5-day period in this curve, and it is less difficult to see how the successive four periods in the first curve may have been produced by accident. No Fourier analysis has been made of the curves Cl − Q, because whatever might be obtained, the interpretation of the result would be doubtful on account of the changing period. It is only when we have a long succession of periods with comparatively small changes of period that the process used in the previous analysis can be usefully adopted.

The most obvious explanation of the term is an oscillation of the pier to which each clock is bolted, which may be communicated from one pier to the other in some manner. If this explanation be adopted, it is necessary to account for the difference of phase. Since the planes of motion of the pendulums of the two clocks intersect in a line which is equally distant from the two clocks, it is necessary to suppose that the piers do not oscillate with the same freedom in all directions.* The magnitudes appear to require amplitudes of oscillation of the order of one or two wave-lengths of light at the points of suspension of the pendulums. Mr. Loomis points out, however, that while these piers have not been directly tested for such motions, other clocks less rigidly mounted have shown movements less than a small fraction of a fringe. A vertical compressional wave would seem to have too small an amplitude to give the effect; it again would require lack of symmetry in the yielding material to explain the phase difference. Calculation shows that the direct attraction of the pendulums on one another is far too small and that it would require the phase difference to be 0° or 180°. The observed phase difference appears to cut out any attempted explanation based on the interference of the electrical connections of the clocks.

The various processes followed in the analysis, a summary of which is contained in this paper, involved a large amount of calculation, much of which was performed by Mrs. D. Brouwer. We felt, however, that a full examination of the records might be useful, since it is the first time that a continuous record has been made of the performance of a clock to a degree of accuracy beyond that of the clock. The expense of the computations was met by the Loomis Institute.

* The horizontal section of the pier of Clock 2 is a square, that of Clock 3 is a right-angled isosceles triangle.

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