Project GREAT (2005)

General Relativity Einstein / Essen Anniversary Test

\text{GREAT}^2

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Washington DC
Introduction

• Project GREAT in 2005
  - Attempt to prove the theory of relativity
  - Take cesium clocks up a mountain
  - Do clocks really speed up or slow down?

• Celebrate 100th anniversary of 1905
  - Albert Einstein’s “Annus Mirabilis”

• Celebrate 50th anniversary of 1955
  - Louis Essen’s NPL cesium clock
Albert Einstein

• Who was Einstein?
  - Need I say more...
  - Theory of relativity
  - Time is not absolute
  - SR, GR, space-time
  - Bold predictions
  - Later confirmed
  - Enormous influence
Einstein and 2005

- 100th anniversary of relativity: books, magazines, radio, TV, web sites, “Physics Year”, lectures...
Louis Essen

- Who was Essen?
  - First Cesium Clock
  - Joint NPL USNO project to calibrate atomic time against astronomical time
  - 9 192 631 770 Hz
  - Book: “Famous for a second”
Essen and 2005

• 50\textsuperscript{th} anniversary of atomic time
• NPL Caesium

Jack Parry and Louis Essen
Photo from www.npl.co.uk/essen/
Cs Second

- 1954...1958
- How long is a second?

comparison between the cesium beam at Teddington and the moon camera at Washington. From an analysis of the various factors involved we have adopted a probable error of ±20 cps.

We find, thus, the transition frequency of cesium (4, 0) → (3, 0) at zero magnetic field is

\[ \nu_E = 9192631770 \pm 20 \text{ cycles per second (of E.T.) at 1957.0.} \]

The mean epoch is specified because there is a possibility that the gravitational and atomic time scales may not be the same, and may change secularly. Future determinations of \( \nu_E \) will decide this question.

| Table I. Results for \( \nu_E \) obtained from four different sets of data. |
|---------------------------------|----------|----------|----------|----------|
| Means                  | \( \Delta T_a \), 1954.25-1958.25 | +1.146 | -121 | 9192631761 | + 0.17 |
| \( \Delta T_0 \), 1955.25-1958.25 | +1.085 | -115 | 767 | + 0.10 |
| \( \Delta T_c \), 1954.25-1958.25 | +1.035 | -110 | 772 | + 0.12 |
| \( \Delta T_c \), 1955.25-1958.25 | +0.966 | -102 | 780 | + 0.17 |
Louis Essen

• 10 years later …
• Essen at NPL with a HP 5060A "Flying Clock"

Fig. 10. Dr. L. Essen, who developed world’s first Cesium$^{133}$ atomic-controlled frequency standard at National Physical Laboratories, examines compact cesium-beam frequency standard that controls traveling clock.
Flying Clocks in the 1960's

- Starting in 1964 with HP 5060A
- Portable transistorized cesium clock
- Hundreds of clock trips
- Remote synchronization to $\mu$s levels
- See HP Journals: 1964, 65, 66, 67
- 1965 world-wide time synchronization
- Paved the way for flying clock relativity experiments in the 1970's
Relativity and Clocks

• High-level summary:
  - Clocks run slower if they move at high velocity (SR)
  - Clocks run slower in the presence of greater gravity (GR)
  - Clocks lose time traveling East (Sagnac)

• This implies:
  - According to general relativity, stationary clocks on mountains run faster.
And so... 2005

- General
  - Relativity
  - Einstein
  - Essen
  - Anniversary
  - Test

- Project GRE\(^2\)AT
Chapter 2

- Flying clock experiments
1971 Hafele & Keating

- PTTI, vol 3, 1971
- Time, Oct 18, 1971
- Science, Jul 14, 1972

Table 1. Observed relativistic time differences from application of the correlated rate-change method to the time intercomparison data for the flying ensemble. Predicted values are listed for comparison with the mean of the observed values; S.D., standard deviation.

<table>
<thead>
<tr>
<th>Clock serial No.</th>
<th>$\Delta \tau$ (nsec)</th>
<th>Eastward*</th>
<th>Westward</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>-57</td>
<td>277</td>
<td></td>
</tr>
<tr>
<td>361</td>
<td>-74</td>
<td>284</td>
<td></td>
</tr>
<tr>
<td>408</td>
<td>-55</td>
<td>266</td>
<td></td>
</tr>
<tr>
<td>447</td>
<td>-51</td>
<td>266</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-59 ± 10</td>
<td>273 ± 7</td>
<td></td>
</tr>
<tr>
<td>Predicted ± Error est.</td>
<td>-40 ± 23</td>
<td>275 ± 21</td>
<td></td>
</tr>
</tbody>
</table>

* Negative signs indicate that upon return the time indicated on the flying clocks was less than the time indicated on the MEAN(USNO) clock of the U.S. Naval Observatory.
Hafele & Keating

- Round-the-world (EW/WE) flying clocks
- Hafele thought there would be effect
- Keating didn’t (was open to finding out)
- Many scientists were “certain”
- Results were stunning: all 4 clocks showed difference between EW and WE
- Relatively simple, cheap experiment
1975 C.O. Alley

- Multiple flights
- P3C airplane
- Chesapeake Bay
- Environment
- Multiple clocks
  - 3 Cs, 3 Rb, 2 HM
  - L.C. 5061A
- Laser link
C.O. Alley

- 3 different altitudes (25, 30, 35 000)
- 5 flights, 2 h, multiple clocks, 0.5%
1976 R.F.C Vessot & M. Levine

- H-maser up, up and away: 10,000 km!
- NASA, ground stations, up/down links
- GP-A amazing results, 0.0070%, 70 ppm
1977 NTS-2

- Navigation Test Satellite #2
- First NAVSTAR GPS, cesium in space
- Verify relativistic effects

Fig. 20 - Cesium frequency via (T-O) slope

Fig. 21 - Effect of relativity correction

Fig. 27 - Frequency offset summary
Recent relativity experiments

• 1996 NPL & BBC - 25th anniversary of H&K with London (NPL) to Washington DC (USNO)
• 2000 T. Celano, TSC - military airplane, cesium clocks, comsat modems
• 2002 T. Celano, TSC - similar but with two-way communication, real-time corrections
Recent relativity experiments

• 2000-2002 - CRL (NICT) measured GR time dilation when moving several 5071A between low and high altitude facilities
• Also, many non-atomic clock relativity experiments are in progress
• GP-B, black holes, LIGO, ...
Chapter 3

• The Big Idea
Relativity at home?

• Theory of relativity well confirmed
  - With planets, particles, atomic clocks
• But is it so extreme, so exotic, that only places like Harvard, USNO, JPL, or NASA can prove it?
• Or is it possible to perform a home experiment to confirm Einstein’s prediction? This seems far-fetched.
Relativity at home

• **Methods; take atomic clock**
  - At high speed, or
  - To high altitude, or
  - On long eastward or westward trip, or
  - All the above

• **Modes; transport using**
  - Airplane
  - Rocket
  - Satellite
Relativity at home

• None of those methods work for me:
  - I don’t have a plane
  - Rocket or satellite is out of the question

• But I do have:
  - Many atomic clocks
  - Nearby mountains

• So, use a mountain?
Northwest Mountains

- (N) Mt Baker, Glacier Peak
- (S) Mt Rainer, Mt Adams, Mt St Helens
- (E) Olympic range, Cascade range…
**Back of Envelope Calculation**

- According to GR, clock frequency changes approximately by:
  \[ \approx g \cdot \Delta h / c^2 \]
- On earth, this is \[ \approx 1.09 \times 10^{-16} \text{/meter} \]
- That's really small!
- Too small for me (or anyone) to measure

From NPL website
1.1×10^{-16} is too small, but

- Say, you go up 1 km instead of 1 m
  \[ \Delta f = 1.1\times10^{-13} = 0.11 \text{ ps/s} \]
- And stay a whole day
  \[ \Delta T = \Delta f \times 86400 \text{ s} = 9.5 \text{ ns} \]
- 9 ns is “huge”; so this looks possible!
- The key to detecting time dilation: go high and stay long
- Sign is + (blue shift)
The Big Idea

• Take our 3 kids and 3 cesium clocks up Mt Rainier
• See if Einstein was right about gravity and time
• See if clocks really run faster up there
Map: Seattle to Mt Rainier

- Just 100 miles away (~2 1/2 hours)
Math Detail

• To a first approximation, small $v$, $\Delta h$

• Kinematic: $\Delta f_k \approx -\frac{1}{2}v^2/c^2$

• Gravitation: $\Delta f_g \approx +g\Delta h/c^2$

• Sagnac: $\Delta f_s \approx -\omega R^2\cos^2(\phi)\cdot\lambda / c^2$

• Net $\Delta f = \Delta f_k + \Delta f_g + \Delta f_s$

• Total $\Delta T = \sum \Delta f \times T$
Prediction: Sagnac Effect

- $\cos^2$ factor is 0.5 for 45° vs. equator
- 200 ns for 40 000 km round-the-world
  - So 0.001x that for 40 km, or 200 ps
- No effect for N-S travel
- And no effect for same-path round-trip
- So ignore Sagnac effect
Prediction: Velocity Effect

- Automobile speeds are relatively low
  - 65 mph ≈ 96 fps ≈ 100 kph ≈ 30 m/s
- Actual trip is 100 miles and 2.5 hours
  - Average speed below 40 mph
- Worst case 30 m/s for $10^4$ s
  - so $\Delta f = 5 \times 10^{-15}$, and $\Delta T = -50$ ps.
- So ignore Velocity factor too!
Prediction: Gravitational

- Guess 40 to 48 hours
- Guess 1640 - 300 = 1340 meters
- $\Delta f = 1.5 \times 10^{-13}$
  - This $\Delta f$ is very measurable with 5071A
- 1340 m $\times$ 40 hours
- $\Delta T \approx +20$ ns
  - This $\Delta T$ is very measurable with 53132A
## Time Dilation Examples

<table>
<thead>
<tr>
<th>Transport</th>
<th>Speed (km/h)</th>
<th>Altitude (km)</th>
<th>$\Delta f$</th>
<th>$\Delta T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walking</td>
<td>5</td>
<td>0</td>
<td>~0</td>
<td>~0 ps/h</td>
</tr>
<tr>
<td>Car</td>
<td>100</td>
<td>0</td>
<td>$-4 \times 10^{-15}$</td>
<td>16 ps/h 0.4 ns/d</td>
</tr>
<tr>
<td>Balloon</td>
<td>0</td>
<td>10</td>
<td>$+1.1 \times 10^{-12}$</td>
<td>+95 ns/d</td>
</tr>
<tr>
<td>Plane</td>
<td>900</td>
<td>10</td>
<td>$-3.5 \times 10^{-13}$</td>
<td>-30 ns/d</td>
</tr>
<tr>
<td>GREAT 2+2 h drive</td>
<td>100</td>
<td>0</td>
<td>$+1.1 \times 10^{-12}$</td>
<td>60 ps</td>
</tr>
<tr>
<td>GREAT 40 h stay</td>
<td>0</td>
<td>1.340</td>
<td>$+1.5 \times 10^{-13}$</td>
<td>20 ns</td>
</tr>
</tbody>
</table>
Summary – Calculations

• “Many, High, Long”
• 3 best clocks (5071A/001)
• 1340 m (5400’ - 1000’) altitude
• Weekend (40 h)
• \(1 \times 10^{-16} \) & 1340 m & 40h & \( \sqrt{3} \)
• Estimate \( \sim 20 \) ns
• Estimated accuracy \( \sim 2 \) ns, 10%
• Time dilating at about 500 ps / hour
Clocks on Mountains

- Climb or drive?
- Plus batteries...
Clocks on Mountains

• Drive, of course
• This is America…
One-way or Round-trip

• Clocks run slower on a mountain so:
  - Measure frequency of clock at home
  - Measure frequency of clock on mountain

• Measure against what?
  - How about GPS time & frequency
  - Need better than $10^{-13}$ accuracy
  - Not with my GPS receivers
GPS reference

- Direct frequency measurement (GPS)
Summary - Big Idea

- 3 kids, 3 clocks, 1 family minivan
  - To 5400 feet
  - For 40 hours
  - Expect +20 ns
- Sync time/rate before trip
- Measure time/rate after trip
- Against reference; my "house standard"
Chapter 4

• Home time lab
Home Time Museum & Lab
Where does it all come from?

- Local aero/mil-surplus electronics
- Ham conventions; flea markets
- Used test equipment dealers
- Demo or refurbished models
- Friends, strangers; other “time-nuts”
- Sympathetic T & F companies
- And, of course, eBay!
Surplus Time & Frequency
Surplus Time & Frequency

HP Agilent 5061B Cesium Beam Frequency Standard w/Opt3

Bidding has ended for this item
If you are a winner, Sign in for your status.

This item or one like it has been relisted.

Starting bid: US $2,999.99
Make no payments for 3 months - App!

Ended: Sep-11-06 14:30:00 PDT
Shipping costs: US $75.00 (discount available)
Standard Flat Rate Shipping Service
Ships to: United States
Item location: China, China

HP Agilent 5061A Cesium Frequency Standard

Bidding has ended for this item at US $405.00. You were outbid.

Increase your chances of winning next time:
- Track your bidding, especially during the final minutes of a listing. Use My eBay email notifications.
- Try entering your highest maximum bid. eBay will only use as much of your bid as necessary.
- Use completed item search to better estimate the final price of items you bid on.

Winning bid: US $405.00
Make no payments for 3 months - App!

Ended: Aug-28-06 13:30:00 PDT
Shipping costs: Calculate
Ships to: United States
Surplus Time & Frequency
Surplus Time & Frequency
Surplus Time & Frequency

Ebay: Frequency Electronics Cesium Beam Standard/ Clock (item 120034340084 end time Sep-27 06 20000000 PDT (1 day 8 hours)
Starting bid: US $999.99
End time: Sep-27 06 20000000 PDT (1 day 8 hours)
Shipping costs: Check item description and payment instructions or contact seller for details
Sells to: United States
Item location: Millersville, Maryland, United States
History: 0 bids

Ebay: Datum Austron GPS Rubidium Clock (item 130021839748 end time Sep-08 06 04:18:28 PE
Starting bid: GBP 700.00
Ends: Sep-08 06 04:18:28 PE
Ships to: Worldwide
Item location: London, London, United Kingdom
History: 0 bids

You can also: Watch this item
Get alerts via Text message, IM or Phone call

Surplus Time & Frequency
Surplus Time & Frequency

FTS 4040A Cesium frequency standard

Bidding has ended for this item.
If you are a winner, Sign in for your status.

Sell an item like this or buy a similar item below.

Winning bid: US $850.00
Make no payments for 3 months - Ask

Ended: Sep. 12, 06 19:00:00 PDT
Shipping costs: Check item description and payment instructions or contact seller for details
Ships to: United States, Canada
Item location: Manchester, New Hampshire, United States

You can also: Watch this item
Get alerts via Text message or IM

Listing and payment details: Show

Stanford Research FS700 Frequency Standard
10 MHz LORAN-C Frequency Standard with Antenna!

You are signed in

Buy it Now price: US $950.00
Buy it Now

Shipping costs: Check item description and payment instructions or contact seller for details
Ships to: Worldwide
Item location: Oceanside, CA, United States

You can also: Watch this item
Get alerts via Text message or IM
Sell one like this

Listing and payment details: Show
Surplus Time & Frequency

FTS 1000 ULTRA STABLE OSCILLATOR FREQUENCY STANDARD

You are signed in

Buy It Now price: US $757.25  Buy It Now

Make no payments for 3 months - App

Shipping costs: US $12.00
Standard Flat Rate Shipping Service

Ships to: Worldwide
Item location: New York, United States
Quantity: 2 available

History: Purchases

You can also: Watch this Item
Get alerts via Text message or IM

View larger picture

FTS 1130 ULTRA STABLE OSCILLATOR FREQUENCY STANDARD

You are signed in

Buy It Now price: US $757.25  Buy It Now

Make no payments for 3 months - App

Shipping costs: US $12.00
Standard Flat Rate Shipping Service

Ships to: Worldwide
Item location: New York, United States

You can also: Watch this Item
Get alerts via Text message or IM

View larger picture
Surplus Time & Frequency
Surplus Time & Frequency
Surplus Time & Frequency
Surplus Time & Frequency

Efracom Rubidium Portable Atomic Clock PC 10

Bidding has ended for this item
If you are a winner, Sign in for your status.

Sell an item like this or buy a similar item below.

Winning bid: GBP 269.90
(Approximately US $412.84)

Ends: Sep 03-06 13:20:06 PDT
Ships to: Worldwide
Item location: Henley-on-Thames, United Kingdom
History: 12 bids
Winning bidder: kaulus (private)

Efracom 10 MHz Rubidium Frequency Standard COMPLETE!

You are signed in

Current bid: US $202.50
Place Bid>

Reserve not met!

End time: 19 hours 8 mins
(Sep 27-06 10:01:51 PDT)
Shipping costs: Check item description and payment instructions or contact seller for details
Ships to: Europe, Australia, Asia, N. & S. America
Item location: Westford Massachusetts, United States
History: 9 bids
High bidder: tmack (42 *)

06-Dec-2006 Project GREAT
Surplus Time & Frequency
Home Time Lab

• Not just dirt cheap clocks, but...
• Manuals, cables, connectors, power supplies, meters, GPIB, USB, software, GPS antennas, GPSDO, WWVB, Loran-C, clocks, displays, IRIG, TCG, frequency counters, TIC, phase comparators, etc.
• Most old, some new; possibly broken, often working. Buy 3, play Frankenstein.
Museum of HP Clocks
Requirements

- GR effect $1.1 \times 10^{-16} / m$
- $\Delta f = 1.5 \times 10^{-13}$
- If we want 10% accuracy
- We need a reference good to $10^{-14}$
Chapter 5

• A Powers of Ten tour
Powers of Ten - Introduction

• A quick overview of clock accuracy
• What clocks keep poor time?
• What clocks keep best time?
• And many in between...
10^-0 drip, drip

- Leak in ceiling
- 0.57 s ... 9.9 s
- 1.7 Hz ... 0.1 Hz
$10^{-1}$ heart beat

- $10^{-1}$, 0.1, 10%
- The original ‘1 PPS’
- Sometimes 2x, even 3x
- Much higher stability at night
- < 10% accuracy possible
$10^{-1}$ heart beat

- 12 h frequency plot (evening/night)
- ADEV floor is $10^{-1}$ from $10^1$ to $10^4$ s!
- (is this OK?)
10^{-2} tuning fork oscillator

- 0.01, 1%
- General Radio Type 213 Audio Oscillator
- 1 'kc'; f = \sim 992.8\,\text{Hz}
- \pm 1.3\,\text{mHz (60 x 1 s)}
- Accuracy < 1%
- Count those 9’s
- ADEV is 10^{-6}
$10^{-2}$ tuning fork oscillator
$10^{-3}$ precision tuning fork

- 0.001, 0.1%, 1 ms/s
- General Radio Type 813
- 1 'kc' tuning fork
- $f = \sim 999.4$ Hz
- $\pm 400 \mu$Hz (60 x 1 s)
- Accuracy < 0.1%
- ADEV is $10^{-7}$
$10^{-3}$ precision tuning fork
10^{-4} mechanical oscillator

- 0.01%, 100 ppm
- Mechanical oscillator
- “Four 9’s”

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Mean (Hz)</th>
<th>Max (Hz)</th>
<th>Min (Hz)</th>
</tr>
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<tbody>
<tr>
<td>999.907,211,67</td>
<td>999.907,159,334</td>
<td>999.907,404,05</td>
<td>999.906,840,54</td>
</tr>
<tr>
<td>999.907,250,33</td>
<td>999.907,311,01</td>
<td>999.907,250,27</td>
<td>999.907,345,09</td>
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<tr>
<td>999.907,273,16</td>
<td>999.907,311,01</td>
<td>999.907,250,27</td>
<td>999.907,345,09</td>
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<tr>
<td>999.907,311,01</td>
<td>999.907,345,09</td>
<td>999.907,250,27</td>
<td>999.907,345,09</td>
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<tr>
<td>999.907,345,09</td>
<td>999.907,345,09</td>
<td>999.907,250,27</td>
<td>999.907,345,09</td>
</tr>
<tr>
<td>N : 60</td>
<td>STD DEV: 151.812 uHz</td>
<td>MEAN : 999.907,159,334 Hz</td>
<td>MAX : 999.907,404,05 Hz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MIN : 999.906,840,54 Hz</td>
<td></td>
</tr>
</tbody>
</table>
$10^{-5}$ mains

- 0.001%, 10 ppm
- $60 \pm 0.001$ Hz

60 Hz Mains Frequency Deviation Histogram
2.7 million one second samples (~1 month)

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Count</th>
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</thead>
<tbody>
<tr>
<td>60.000, 640, 720, 7</td>
<td>1</td>
</tr>
<tr>
<td>60.009, 491, 393, 8</td>
<td>1</td>
</tr>
<tr>
<td>60.000, 431, 181, 6</td>
<td>1</td>
</tr>
<tr>
<td>59.992, 198, 219, 9</td>
<td>1</td>
</tr>
<tr>
<td>59.987, 371, 509, 5</td>
<td>1</td>
</tr>
<tr>
<td>59.993, 148, 200, 6</td>
<td>1</td>
</tr>
<tr>
<td>59.999, 032, 462, 5</td>
<td>1</td>
</tr>
<tr>
<td>59.985, 892, 634, 1</td>
<td>1</td>
</tr>
<tr>
<td>59.995, 727, 396, 2</td>
<td>1</td>
</tr>
<tr>
<td>N = 36</td>
<td></td>
</tr>
<tr>
<td>STD DEVI: 0.006, 765, 596, 40 Hz</td>
<td></td>
</tr>
<tr>
<td>MEAN: 59.999, 554, 563, 23 Hz</td>
<td></td>
</tr>
<tr>
<td>MAX: 60.010, 390, 980, 5 Hz</td>
<td></td>
</tr>
<tr>
<td>MIN: 59.985, 892, 634, 1 Hz</td>
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<tr>
<td>59.996, 011, 518, 6 Hz</td>
<td></td>
</tr>
<tr>
<td>59.999, 508, 109, 7 Hz</td>
<td></td>
</tr>
</tbody>
</table>
$10^{-5}$ mains

**Phase Data**

- Frequency vs. Time
- Data: 06-Dec-2006
- Project: GREAT

**Frequency Stability**

- Modified Allan Deviation, $\sigma_y(\tau)$ vs. Averaging Time, $\tau$
- Data: 06-Dec-2006
- Project: GREAT
$10^{-6}$ quartz watch (RC)

- 0.0001%, 1 PPM, 1 $\mu$s/s
- +160 ms/d = +1.85 ppm
10^{-6} quartz watch (RC)

- Nightly WWVB radio sync (60 kHz)
- Look closely at 01:30 AM PST
- +1h +30m +15s
- Plot of 9 days
- Rate variations
- Sync variations
Aside: Quartz Wall Clock

- Quartz crystal and divider/driver IC
- Stepper motor (180° per step)
Quartz Wall Clock

- 32 kHz oscillator
- 1 Hz stepper
Quartz Wall Clock

- Polarity alternates
- Pulse size: 1.5 V x 50 ms
Quartz Wall Clock

- **Coil current**: $1.5 \, V / 500 \, \Omega = 3 \, mA$
- **Oscillator current**: $<1 \, \mu A$
- **Pulse power** ($V \times A$): $4.5 \, mW$
- **Pulse width**: $50 \, ms$
- **Clock Energy** ($P \times T$): $4.5 \, mW \times 50 \, ms$
  \[= 225 \, \mu J = 60 \, pico \, kWh\]
- **AA battery** ($2850 \, mAh$) = $\sim 2 \, years$
$10^{-7}$ chronometer

- 0.1 ppm
- Rated $\frac{1}{4}$ sec/day deviation
10^{-7} chronometer

- ~55 hour runtime
- 200 ms phase residuals
- ADEV 6 \times 10^{-7}
10⁻⁷ chronometer

- From 1940’s USN manual...
- Phase
  - Dial error
- Frequency
  - Daily rate
- Drift
  - Deviation in rate

### Table: Computation of Rate

<table>
<thead>
<tr>
<th>Date</th>
<th>Dial Error</th>
<th>Daily Rate</th>
<th>Mean Deviation in Daily Rate</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>Sec</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>01 4/40</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>+0</td>
<td>2</td>
<td>+1/2</td>
<td>Started+Set</td>
</tr>
<tr>
<td>4</td>
<td>+0</td>
<td>2 1/2</td>
<td>+1/2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>+0</td>
<td>2 1/2</td>
<td>0</td>
<td>1/4</td>
</tr>
<tr>
<td>6</td>
<td>+0</td>
<td>3</td>
<td>+1/2</td>
<td>1/4</td>
</tr>
<tr>
<td>7</td>
<td>+0</td>
<td>3</td>
<td>0</td>
<td>1/4</td>
</tr>
<tr>
<td>8</td>
<td>+0</td>
<td>3 1/2</td>
<td>+1/2</td>
<td>Not wound</td>
</tr>
<tr>
<td>9</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>+0</td>
<td>4</td>
<td>+1/4</td>
<td>2 day avg</td>
</tr>
</tbody>
</table>

(Mean daily rate = +1/4 second)

In Table I, there will be noted a column headed "Mean Deviation in Daily Rate." The
$10^{-8}$ pendulum clock

- 0.01 ppm, 10 ppb
  10 ns/s, 864 μs/d
- Shortt, Fedchenko, Riefler, 'Littlemore'
$10^{-8}$ pendulum clock

- Amazing astronomical pendulum clocks
- Several centuries of understanding and perfection. Limitations addressed:
  - Temperature, humidity, mass, friction, metallurgy, escapement, master/slave, vacuum, isochronous suspension, etc.
- When all factors solved, the best pendulum clock is just a good gravimeter
$10^{-8}$ pendulum clock

Littlemore, Shortt, and 'Perfect' Pendulum Clock

Allan Deviation (overlapping)

Averaging Time, Tau (seconds)
$10^{-9}$ earth

- 0.001 ppm
- Slow by $\sim 2$ ms per day
- Also somewhat irregular
- ADEV $10^{-8} \sim 10^{-9}$
- Limited by core, weather, climate
- Also lunar/solar tides
$10^{-9}$ earth
10^{-9} \text{ earth}

- Earth as a frequency standard
- Suggested improvements:
  - Thoroughly clean, and dry with cloth
  - Remove surrounding gas and water vapor
  - Wait for core to cool before use
  - Re-align axis of rotation (wobbling)
  - Keep away from nearby moon (tides)
  - Keep away from sun (tempco)
  - Re-adjust rate (avoid leap seconds)
10^{-10} \text{ ocxo}

- 0.1 ppb, 100 ps/s, 8.64 \mu s/d
- $10^{-10} \ldots 10^{-13}$ short
- $5 \times 10^{-10}$/d drift

Allan Deviation $\sigma_y(\tau)$

Ch A: 5.0 MHz 2.7 V_{pp}
Averaged Phase

Ch B: 10.2 MHz 1.4 V_{pp}
B/A=2.04580007663435
$10^{-11}$ good ocxo

- 0.01 ppb, 10 ps/s, 864 ns/d ($\sim 1 \mu$s/d)
- $10^{-11}$...$10^{-13}$ short
- $\sim 10^{-11}$/d drift

Allan Deviation $\sigma_y(\tau)$

Ch A: 5.0 MHz 2.0 V$_{pp}$
Averaged Phase

Ch B: 5.0 MHz 1.6 V$_{pp}$
B/A=Single DDS
$10^{-12}$ excellent ocxo

- 1 ppt, 1 ps/s, 86.4 ns/d (~100 ns/d)
- $\sim 10^{-13}$ short/mid
- $\sim 3 \times 10^{-12}$/d drift
$10^{-13}$ rubidium

- 8.64 ns/d ($\sim 10$ ns/d)
- $\sim 10^{-13}$ mid-term
- $\sim 1 \times 10^{-11}/m$ drift
\(10^{-14}\) cesium

- 864 ps/d (~1 ns/d)
- \(\sim 10^{-13}\) mid-term
- \(\sim 1 \times 10^{-14}\) @ 1 day
10^{-14} more cesium

• 10^{-14} not!
• Cesium clocks differ by 2x - 50x
• E.g., old 5060A vs. new 5071A

![Image of a cesium clock]

![Graph showing Allan Deviation]
10^{-14} another cesium

- Not even close to 10^{-14} @ 1 day
- FTS 4010
- Portable Clock
- Old, tired
one more cesium

- About $2 \times 10^{-13}$ @ 1 day
- FTS 4050
- See variety of Cs beam cavities
$10^{-14}$ BVA quartz

- But can you get to high altitude and measure in 1 to 100 seconds?
- $10^{-13} \ldots 10^{-14}$ short
- $10^{-11} \ldots 10^{-12}$ drift
10^{-15} active h-maser

- 86.4 ps/d
- Near $1 \times 10^{-15}$ @1d
- Cavity auto-tuned
10$^{-15}$ cesium, long-term

- High-perf units
- Pair near 2$\times$10$^{-14}$ at a day
- Floor near 5$\times$10$^{-15}$ in weeks
Powers of Ten - summary

• $10\%$ to $10^{-15}$ - 15 orders of magnitude
Chapter 6

- Experimental setup
Key Parameters

- How high
- How long
- How stable
- How many
- How precise you measure

Cartoon by Dusan Petricic
Scientific American column Wonders by Philip and Phyllis Morrison
Ingredients

• Portable clocks, *hp 5071A, @3*
  - Check against base clock (home)
  - Check against themselves (away)

• Base clock (passive H-maser)
  - Check against active H-maser)
  - Check against GPS @2

• Measurement system
  - *hp 53132A TIC (~150 ps) @3*
  - Laptop
Portable Clock(s)

- Many Cs tested (5060, 5061, 4050, 4060)
- Three good 5071A/001 chosen
- Color coded clocks, cables, TIC, kids
- Deliberate phase offset of +1, +2, +4 μs
- 1 PPS cables kept with clocks
- HP 53132A TIC per clock
- Data logged at 1 Hz; 5 min averages
5071A

• Pick best ones...

Typical 5071A Cesium Standard Stability
4 std-perf (circle) and 4 high-perf (square)
Data collection

- RS232 interface
- PC serial port logger
- MJD time-stamps
- 8 port serial-to-USB
- 53132A ti, mean, sdev
- 5071A stats
- GPS NMEA

```
scpi > syst:print?

MJD      53630.228067
Status summary: Operating normally
Power source: DC
Log status: 20 entries

Freq Offset: 0e-15 Osc. control: -8.40 %
RF Amplitude 1: 20.2 % RF Amplitude 2: 20.6 %
RF Center Freq: 9999.99 MHz C-Field curr: 12.104 mA
E-multiplier: 1226 V Signal Gain: 14.4 %

CCT Oven: 7.6 V CCT Oven Err: 0.02 °C
Osc. Oven: -9.0 V Ion Pump: 0.00 mA
HW Tunnel: 1.0 V Mass spec: 12.8 V
SW Tuning: 0.0 V Cab: Tuning: 6.2 V
87 MHz PLL: 0.9 V Up Clock PLL: 3.0 V
+12V supply: 12.3 V +12V supply: 12.3 V
+5V supply: 5.5 V Thermometer: 41.1 °C
```
Base Clock, 'House' standard

- Master 1 PPS, CH1-76 passive H-maser
- 1PPS distribution system (8x)
- 1PPS vs. HP 58503B GPSDO
- 1PPS vs. CNS-II (sawtooth-less M12+)
- 1PPS used as ref for 5071A (x3)
- Deliberate phase offset of -4 μs
- 5 MHz vs. active maser
AC / DC power

- Engine tap from minivan 12 VDC
- 2 batteries in parallel for 12 V
- 4 batteries in series/parallel for 24 V
- 12VDC/120VAC inverters
- AC power for clocks
- AC power for laptop(+batt) and TIC
- Triple backup for clocks (AC,DC,+batt)
- Power, voltage, and current monitors
Comment on Clock Accuracy

- Need to predict time 2 days in future
- Not accuracy, but stability
  - ADEV ($\tau=2d$)
- 5071A/001 is $\sim 1 \times 10^{-14}$
- Three clocks:
  - Redundancy (single point of failure)
  - Self-checking (one clock, two clocks...)
  - Lower uncertainty ($\sqrt{3} = 1.7x$ better)
Carrying the Time

• How to measure clocks when at home?
  - Compare with house reference
  - Compare amongst themselves

• How much to trust clocks when away?
  - What do clocks do when you’re not looking?
  - Guess future behavior = known past
  - Past statistics give future predictions
  - ‘Certainty’ replaced with ‘confidence’
Stability: Measurements

- **Phase measurements**: $p_0, p_1, p_2, \ldots$
- **Frequency calculations** ($\tau_T$):
  \[
  f_1 = (p_1 - p_0) / T, \quad f_2 = (p_2 - p_1) / T, \ldots
  \]
Stability: Predictions

- Predict (extrapolate) future phase:
  \[ p_2' = p_1 + f_1 \cdot T = p_1 + (p_1 - p_0) \]

- Later, check prediction error:
  \[ e_2 = p_2 - p_2' = p_2 - 2p_1 + p_0 \]
Stability: Allan Deviation

- Stddev of prediction errors gives you estimate of future accuracy of clock

- This $\Sigma[p2 - 2p1 + p0]$ thing is ADEV
Map - home clocks
Map - mobile clocks
Map - mobile power
Map - Paradise, Mt Rainier
Chapter 7

• Photos of the trip
The GREAT Trip, day 1

• Carrying clock downstairs. Limited time; car is a mess, but it works.
The GREAT Trip, day 1

- Kids in the rear, clocks in the middle, and instrumentation in the front.
The GREAT Trip, day 1

- Dad making final clock BNC connections to TIC; Mom says goodbye.
The GREAT Trip, day 1

- Detail of TIC's and laptop in front seat and clocks in middle seat. 23:33:48 UTC
The GREAT Trip, day 1

• Kids drink stop costs me $8 and 125 ps ($1/4 \text{ hour} \times 500\text{ps/h}).
The GREAT Trip, day 1

• Final gas stop and evening arrival in Rainier National Park.
The GREAT Trip, day 2

• Paradise Inn is at 5400’ elevation. Large parking lot to hide in.
The GREAT Trip, day 2

• Classic old Northwest inn; you should visit sometime.
The GREAT Trip, day 2

• Wonderful hiking trails and climbing.
The GREAT Trip, day 2

- Good, the car is still there. Hike to Glacier Vista (6300')
The GREAT Trip, day 2

• Oh no. The sun is really strong and the A/C isn't working as well as I hoped.
The GREAT Trip, day 2

- Avoid a ticket and move the car again. Ouch, running low in fuel. Now what.
The GREAT Trip, day 2

• Kids are fine. Trip is long. Looking for GPS location for next time. What! Valet shuts car off for an hour.
The GREAT Trip, day 2

- Air is thin; little sleep; ponder Time; spend an hour; why are we doing this; Harrison; GPS. That’s 500 ps/beer.
The GREAT Trip, day 3

- Got gas at 6 AM. Used 15.78 gal in 34 h = 0.46 gph; ~2h/gal, so about 1 ns/gal. Cost me $51 and 745 ps.
The GREAT Trip, day 3

• More hiking, exploring, playing. It’s a fun place for a while.
The GREAT Trip, day 3

• 42 hours is up; time to leave. We’re all tired. Can this really work? Go home.
The GREAT Trip, day 3

• Carry clocks & TIC’s back inside, reconnect same cables, resume phase comparison, unpack car. Sleep.
Chapter 8

• Analysis of GPS data
GPS Log

- Serial NMEA data stream (0.5 Hz)
- $GPGGA (time/lat/lon/alt)
- $GPRMC (time/date/lat/lon/speed)

53632.503380 $GPRMC,120454,A,4644.1107,N,12151.8606,W,26.8,272.6,190905,17.8,E,A*03
53632.503380 $GPGGA,120454,4644.1107,N,12151.8606,W,1,03,2.2,727.7,M,-18.8,M,,*77

- Software tool calculates time dilation

53632.503380 46.7352,-121.8640,728,14,0 17.325260 -0.027115 -0.076591 17.222

- Generated plots
Plots from GPS Log

- Latitude, Longitude

![GPS Log](image1)

![GPS Log](image2)
Plots from GPS Log

- Altitude, Velocity
Predictions from GPS Log

- SR (velocity): 50 ps
- Sagnac effect: ±150 ps (net 1 ps)
Predictions from GPS Log

- GR (gravitational): 22.37 ns
Composite plot - beginning

- Trip start

![Graph showing Project GRE²AT Trip Elevation](LeapSecond.com/great2005)
Composite plot - middle

- More gas
Composite plot - end

- Trip end

![Graph showing Project GRE²AT - Trip Elevation](LeapSecond.com/great2005)
Composite plot (net 22.32 ns)
Chapter 9

• Analysis of clock data
Worked example

- Raw data
- 3 clocks
Worked example

- Red only
- $5.85 \mu s$
- $5.55 \mu s$
- -220 ns
- 10 days
Worked example

- **Slope** (freq)
- **Pre/post**
- $-21.2$ ns/d
- $2.454 \times 10^{-13}$

(2b) Raw Phase Data: **Red Clock**
(3 pre-trip days + 2 trip days + 5 post-trip days)

$y = -0.0212x + 1144.2626$
$R^2 = 0.9980$

$y = -0.0209x + 1126.4509$
$R^2 = 0.9984$
Worked example

- Remove fixed offset ~5.8\mu s

(3) Phase Data: Red Clock
(3 pre-trip days + 2 trip days + 5 post-trip days)
Worked example

- Line fit
- $0 \times 10^{-13}$
- Phase offset
- No freq diff

(4a) Phase Data: Red Clock
(3 pre-trip days + 2 trip days + 5 post-trip days)
Worked example

- More fit
- $1.62 \times 10^{-13}$
Worked example

• Best fit
• $2.43 \times 10^{-13}$
Worked example

- Expand scale
- ~1 ns/d match
- Huge time jump!
Results

• Raw clock phase, with mean
Results

- Red
- 20.3 ns

Project GREAT - Single Clock - Red
3 (pre) + 2 (trip) + 9 (post) = 14 days

Date (MJD) 13-Sep to 29-Sep-2005
Results

- Green
- 17.5 ns
Results

- Blue
- 26.3 ns
- 29.7 ns

Project GREAT - Single Clock - Blue
3 (pre) + 2 (trip) + 9 (post) = 14 days

Date (MJD) 13-Sep to 29-Sep-2005
Results

• Mean
• 23.2 ns
Results

- Composite
- ±4 ns(?)
Analysis Methods

- Frequency before extrapolated forward
- Frequency after extrapolated backward
- Both before and after; mean frequency
- Continuous or hourly/daily averages?
- One day; or ±3 days; -n +m days?
- Mean phase or last/first phase?
- Calculated removal of phase jump until optimal least squares fit?
Phase Jump Simulation

- Artificially remove time dilation
- Stddev improves to a point

![Graphs showing Phase Jump Simulation results]
Ambiguity

• Gap in time is clear, but
• Precise magnitude depends on both pre- post-trip rate precision, which is
• Influenced by frequency averaging time
  – 1 hour? 1 day? 2 days? 7 days?
• Might be a bit subjective, but
• All methods seem to agree to a few ns
Ambiguity

- 3+3?
- 3+5?
- 3+10?
- 1+1?
- 3+3?
Chapter 10

- Base & portable clock performance
Base Clock

• House ref is passive H-maser
• Ref vs. GPS; phase (12-days)
Base Clock

- Ref vs. GPS, ADEV (decade)
- Near/below $10^{-13}$ at 1 d
Base Clock

- Ref vs. GPS, ADEV (*many tau*)
- Note 1 d, 2 d better than 1.5 d
Base Clock

- Composite ADEV
Base Clock

- Passive H-maser ref vs. active H-maser
- $2 \times 10^{-15}$ at 1 day means no worries
Base Clock

- But weird short-term stability
- Probably OK (20 ps jumps, 10 ps noise)
Portable Clock(s)

• ~2 days before trip + ~10 days after trip

5071A Performance (12 days)
Portable Clock(s)

• ~3 week trip would work even better

---

R G B 5071A vs. Maser ref. (12 days around trip)
(200 day R vs. G 5071A in black and \(\pm\sqrt{2}\))

![Graph showing Allan Deviation vs. Averaging Time for different time scales (5m, 1h, 1d, 1w, 2w, 4w, 2m).](image)
Confidence Summary

- Base clock is good to \( \sim 2 \times 10^{-15} \)
- Portable clocks are good to \( \sim 2 \times 10^{-14} \)
- Relativistic effect is \( \sim 1.5 \times 10^{-13} \)
- GPS log is much better than 1%
- No show-stopper glitches
- So experiment accuracy is \( \sim 15\% \)
- 15\% of 22 ns is \( \sim 3 \) ns
3-Hat, phase (home)

- $C_{s_i} - C_{s_j}$ via lab reference
3-Hat, phase (away)

- $C_{s_i} - C_{s_j}$ via mutual-comparisons
3-Hat, phase (combined)

- $C_{s_i} - C_{s_j}$ continuous
3-Hat, resid (home)

- $C_{s_i}$ - $C_{s_j}$ via lab reference
3-Hat, resid (away)

- $C_{s_i} - C_{s_j}$ via mutual-comparisons
3-Hat, resid (combined)

- \( C_{s_i} - C_{s_j} \)
Final Graph; 3+2+3 days

Kids, Clocks, and Relativity on Mt Rainier
Three Cesium Clocks: Red Green Blue & Mean

Residual Phase (ns)

Elapsed Time (days)
Final Graph; +kids +mountain
Chapter 11

• Conclusion
What went wrong

- Difficult physical clock transfers
- Direct morning sunlight temperature
- Unexpected out-of-gas event, OK
- Hotel turned minivan off, OK
- GPS battery ran out, OK
- One TIC was 53131A (500 ps vs. 150 ps)
- Minivan A/C has poor algorithm
- Parking lot hassles, OK
- More exhausting than expected
What went right

• No loss of clock
• No loss of any data
• Relativity effects obvious and stunning
• Three clocks was good idea
• Mean closer than hoped for
• Standard deviation wider than expected
• Good input for another run
• Not bad for a first try
• Kids had a great time!
Conclusions

• Experiment worked rather well
• We now believe what we know
• Fun technical challenge
• Echo of historical experiments
• Useful precise time teaching example
• A fitting 2005 PTTI celebration
• Came back much* older and wiser
• Relativity is now “child’s play”
• Best 22 ns of my life!
Thank you

• For years of inspiration and support
• More info: www.LeapSecond.com
• Email: TVB@leapsecond.com
• Patience from wife & kids
• Time for questions…
Kids, Clocks, and Relativity on Mt Rainier
Three Cesium Clocks: Red Green Blue & Mean
Chapter

• Extra material
Einstein & Atomic Clocks

• Verifying GR with atomic clocks
• Did Einstein know? (he died in 1955)
• NBS Cs 1953, Essen 1955, ...
• Naumann & Stroke article
Project GREAT 2007?

• Make minivan a lab not just a transport
• 2+ week pre-trip, 1+ week post-trip
• Single switch: ext Maser vs. int Rb ref
• Careful solar and thermal insulation
• Use park/Inn power instead of car engine
• Proper air con solution in vehicle
• Real-time plots; wireless status & alerts!
• More kids; more clocks (5?)
• Test 1970’s H&K-era 5061A’s for contrast
• Try direct GPS frequency measurement
Humor – cesium wristwatch

• Bill’s watch...

“Atomic Bill” – first true atomic wrist watch
http://www.leapsecond.com/pages/atomic-bill/
Humor - portable cesium clock

• Tom’s backpack (Project GREAT, ver 1)

“Atomic Tom” – climbing with atomic clocks
http://www.leapsecond.com/pages/atomic-tom/
Humor - ‘Glimmer Twins’

• GR says high clocks run faster
• 1965: Keith gets ‘high’ (for 40 years)
• 2005: looks much older…
Humor - relativity

- Time flies while you’re having fun
- Stay young: fly fast and low
- How to spend more time with your kids
- How to make your wife younger
Humor - make time go faster

- Charlie and the Chocolate Factory
Chapter 1

• Introduction